FACULTAD DE CIENCIAS ECONÓMICAS Y EMPRESARIALES
Grado en Economía y Finanzas

A COMPREHENSIVE RISK MANAGEMENT REVIEW FOR FINANCIAL INSTRUMENTS USING DIFFERENT VALUE AT RISK APPROACHES: WHICH METHODOLOGY IMPROVES MARKET RISK VALUATION?

Trabajo de Fin de Grado presentado por:

ÁNGEL ALCAIDE RICO

CARLOS PADRÓN PÉREZ

Dirigido por:

ANTONIO MARTÍN ARROYO

Departamento de Análisis Económico: Economía Cuantitativa

Madrid, Junio del 2020

ABSTRACT

In recent years, there has been an incrementing need within the financial industry to make use of more sophisticated models to quantify the associated risk in any investment or financial activity, with the goal of achieving an adequate risk management and control in decision-making processes. Accordingly, throughout this academic research project, we present a review of the different methodologies surrounding the Value at Risk framework, one of the most common tools in financial risk analysis and quantification. We perform a deep analysis from standard approaches for measuring VaR to the more complex techniques. We will also review some backtesting procedures used to evaluate VaR models. Therefore, the main focus of this research will be to implement a theoretical and practical analysis of several value at risk methodologies and discuss their respective behaviour under real life scenarios including low but also high volatility periods, such as the one fostered by the recent Covid-19 pandemic. To carry out the investigation, historic data from the daily log returns of the Dow Jones Index will exploited through the open-source software R. Results in this paper suggest that the GARCH (1,1) model parametric approach to VaR is the best method for forecasting VaR, especially under the Student-T distribution assumption of returns. The Historical Simulation non-parametric approach, as well as the Moving Average Volatility model, also had promising results under relatively stable circumstances, but showed their weaknesses when the situation changed and volatility in financial markets dramatically increased as a consequence of the current health crisis. For its part, empirical literature highlights the lack of accuracy of the traditional Riskmetrics methodology, fact that was also observed here, where we obtained very discouraging results under such approach. Lastly, it seems that the Extreme Value Theory significantly underestimated risk, resulting in a surprisingly bad performance.

KEYWORDS

Market Risk, VaR, Non-Parametric, Parametric, Extreme Value Theory, Backtesting

TABLE OF CONTENTS

1. INTRODUCTION

1.1. Risk management growing importance since the financial crisis

Before anything, it is crucial to understand the notion of risk. According to Roszkowski (2010), the concept of risk can be explained as the uncertainty that exists as to what the eventual outcome will be and, as so, it arises in any given situation where there is some doubt about at least one of the possible outcomes during a stated period of time. Hence, the risk inherent in any particular setting depends on the range of possible results and the likelihood and value of each of them. In financial terms, risk could be defined as the possibility that the actual return on an investment will be different from its expected return.

Prior to the wake of the 2008 financial crisis, the apparent economic splendour together with the financial regulatory schemes established since the first Basel Accords issued in 1988 contributed to high market expectations. In such context, the smooth functioning of the financial system seemed guaranteed. Nonetheless, the liberalization and globalization processes that surrounded financial markets exponentially increased the sources of risk of financial instruments, whose behaviour became much more unpredictable and complicated to understand due to their increased grade of sophistication. Unfortunately, the Lehman Brothers´ default announced on 15th September 2008 tragically revealed the financial system weaknesses after a long period of blind commitment to the virtues and soundness of it. Apart from the devastating worldwide economic consequences, this tragic event also represented a major breakdown on investors risk-taking behaviour and left a durable scar on risk perception.

As a matter of fact, in such circumstances, the need for a stricter and tighter financial regulation to avoid future financial catastrophes became evident. The financial system regulation reviews that were carried out since then with, for instance, Basel II, Basel III or Basel IV Accords, were based on a gilding principle: the reinforcing of supervisory frameworks and risk assessment in order to achieve a higher degree of consumer's protection and to avoid the breakdown of the named "too-big-to-fail" companies whose defeat would have disastrous effects for the global economic and financial system. In sum, one of the main challenges posed by the 2008 financial collapse was the development of a strong risk culture in which risk management tools and practices will be given a central role (Ernst & Young, 2013).

1.2. Background of VaR methodology

From now on, the core purpose of this research will entirely be centred around the idea of market risk estimation. Market Risk can be defined as the risk of incurring losses in the portfolio valuation arising from movements in market prices. It includes equity risk, interest rate risk, currency risk, commodity risk and volatility risk (Tsay, 2013).

In that respect, the concept of Value at Risk (VaR) comes from the need to quantify with a certain level of significance or uncertainty the amount or percentage loss that a particular portfolio could face over a certain period of time. In short, VaR models are used to provide a measure of risk exposure to all factors that market risk is concerned with, that is, changes in stock prices, interest rates, foreign exchange rates or commodity prices (Tsay, 2013). This idea was originally proposed in the beginning of the 1990s when several financial institutions such as J.P Morgan tried to search for measures that would quantify the market risk exposure of a firm´s investment. Nevertheless, since then, new approaches to calculate VaR in a more accurate way were presented by other experts. And, as so, there are nowadays several different ways of computing VaR, each of them with its strengths and weaknesses.

As we stated above, the significant trading book losses that banks incurred during the 2008 global financial disaster highlighted the need for the Basel Committee to improve the global market risk scheme. As a response, the Basel Committee introduced at first the Basel 2.5 framework which increased the overall level of capital requirements of financial institutions, and VaR was established as the primarily used methodology to determine such capital requirements. Therefore, apart from being the most important internal standard risk measure at financial institutions nowadays, VaR was also initially chosen as the international standard for external regulatory purpose. Later on, and with the increasing debate around VaR efficiency, Basel framework also suffered important reforms.

The most important advantage of VaR methodology as compared to the models that were previously used resides in its simplicity of understanding and its universal use. Indeed, VaR result is expressed in the unit of measure more intuitive and easier to understand, i.e. maximum potential loss of money. Other measures are expressed in much less clear-cut units such as an average cash flow period (Franco Arbeláez & Franco Ceballos, 2005). In addition, another positive aspect about VaR is that it is a comprehensive modelling of the multivariate risk factor process for different time horizons. Hence, such model is applicable, not only to symmetrically or normally distributed loss functions, but also to portfolios with non-linear instruments which could behave in significantly different ways, like options or derivatives.

Nevertheless, even if VaR is a widely used risk management tool, it is crucial to know its limitations for establishing an effective quantification of risk. Ultimately, the fact that VaR is considered a better risk measure when compared to the previous existing ones does not necessarily mean that VaR is the best possible measure, or not even a sufficiently good one. Furthermore, as we previously mentioned, not all approaches to VaR calculation are equal and rely on the same assumptions. Surprisingly, there are few empirical studies that attempt to compare VaR measures. So, the main focus of our work will be to try to make our contribution to this field by answering to the following question: to which degree does each different approach of VaR methodology improve market risk valuation?

On a first step, we will introduce all the theoretical background surrounding VaR methodology. This includes a presentation of the mathematical definition of such concept as well as its statistical goodness properties. Finally, in order to introduce the next step of our work, the different existing models of value at risk will be briefly presented.

On a second step, we will carefully theoretically explain the wide range of methods chosen. Furthermore, we will compute VaR for all such different approaches, for which a detailed description and analysis of the programming codes borrowed from Tsay´s book: *An Introduction Analysis of Financial Data* with R will be provided. In this stage, we will take data from one of the world's most widely followed indexes: The Dow Jones Index, and we will examine its behaviour in a large period of time, which includes both stable phases and volatile phases such as the one we are currently facing with the Covid-19 pandemic.

Finally, in order to reach the final goal of our project, we will test the performance of the different models of VaR used according to the results obtained. This work will be done both quantitatively and qualitatively using the available literature together with accurate back-testing statistical methods.

2. THEORETICAL DISCUSSION OF VALUE AT RISK

The purpose of this section is to present the theory behind the VaR concept in order to provide a solid knowledge basis for the understanding of the logic of such methodology. Additionally, the different existing approaches to VaR calculation will be shortly presented and classified into broad categories for a better arrangement of the future sections of our work.

2.1. Statistical Theory and Factors involved behind VaR calculation

As it has been mentioned already, value at risk has become the standard measure that financial analysts use to quantify market risk. It could be defined as the maximum potential loss in value of a portfolio due to adverse market movements, for a given probability. Even if the statistical definition of VaR does not include very complex mathematical concepts, a proper understanding of such notions is fundamental.

Therefore, let us first revise some important terminologies in statistics and probability theory. First, a random or stochastic variable is described as a function that assigns a numerical value to each elementary event of the sample space. In addition, this variable is said to be continuous if it can take an infinite and so uncountable number of possible values. Furthermore, the probability distribution function of a random variable is a function that describes the relative likelihood for this random variable to take on a given value. Hence, all the information about a random variable X is contained in its probability density function. More precisely, if a random variable X has a continuous distribution function, the probability density function of X is defined as a positive function f(x) and needs to satisfy the following two conditions:

i. $f(x) \geq 0, \ \forall x$

ii. $\int_{-\infty}^{+\infty} f(x)dx = 1$

Finally, all random variables have a cumulative distribution function which could be defined as follows. If X is a continuous random variable taking infinite number of values and with density function f(x), the cumulative distribution function of X, represented as F(x), is a function that assigns to each real value x the probability of X having values less than or equal to x. That is:

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)dx$$

After this small recapitulation of some key concepts of probability theory we are in position to introduce the mathematical definition of value at risk. We need to assume that the rate of return with respect to the time horizon of an investment is a random variable with its respective distribution function. Hence, quantitatively, it is possible to determine VaR using the loss random variable of a financial position for a given period of time. Let´s suppose that at the time index t we are interested in the risk of a financial position for the next h periods. Let $L_t(h)$ be the loss random variable function over the period from t to t+h. $L_t(h)$ takes either negative or positive values depending on the financial returns: for negative financial returns the function will take positive values and vice versa. Also, denote the cumulative distribution function of $L_t(h)$ by $F_L(x)$. Then, for a given probability p, the value at risk is the smallest value x such that CDF $F_L(x)$ of the loss function $L_t(h)$ would be higher to 1-p. It can be expressed in this way:

$$\mathrm{VaR}_{(1\text{-}p)} = \inf\{x: F_L(x) \geq (1-p)\} \text{ and so } F_L(\mathrm{VaR}_{(1\text{-}p)}) \geq 1\text{-}p$$

Therefore, the probability that the position holder would encounter a loss greater than $\mathrm{VaR}_{(1\text{-}p)}$ over the period from t to t+h is p. This previous definition shows that VaR is concerned with the upper tail probability of the CDF of $L_t(h)$ and so, we could alternatively define it as the $(1\text{-}p)^{th}$ quantile of the probability distribution of the loss random variable of financial returns. This means that 100(1-p) % of the values taken by the loss random variable are below VaR and only 100p % of the values are above it. For a better comprehension of the concept, VaR has been illustrated graphically in Figures 1 and 2 below.

Figure 1. Definition of VaR through the CDF of the loss random variable. Tsay (2013).

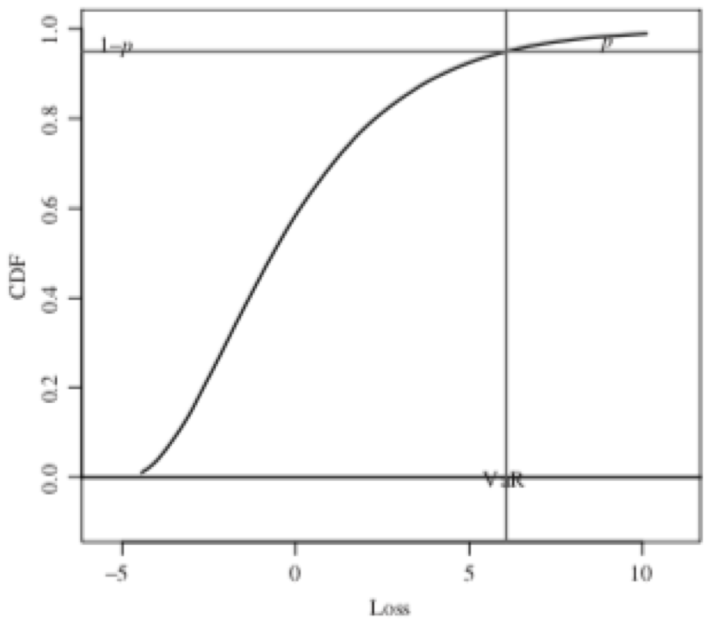
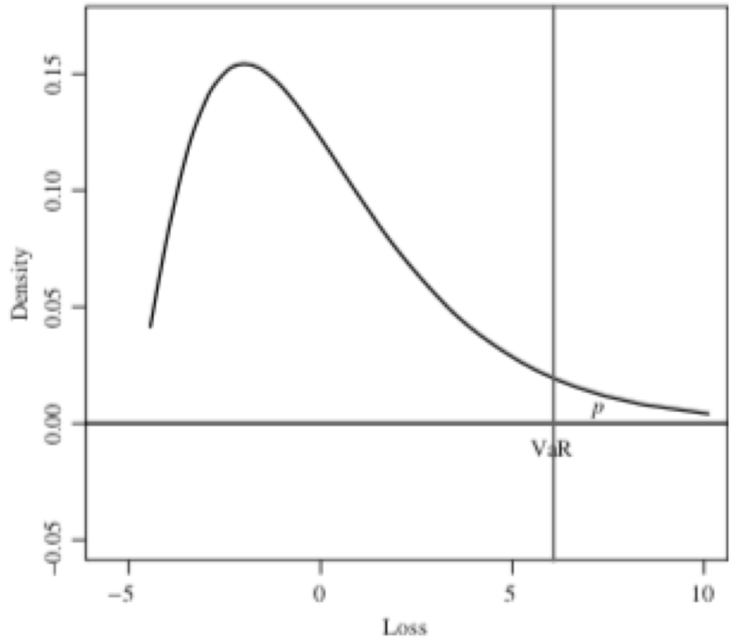


Figure 2. Definition of VaR through the PDF of the loss random variable. Tsay (2013).



From this mathematical definition of the VaR concept it can be derived that such value could be interpreted as follows: The highest amount of dollars/euros or any other currency I can expect to lose over the next h periods of time t is x, with a 1-p level of confidence. Besides, note that we can begin to find here one of the highest disadvantages of VaR methodology. In fact, as we see, VaR does not describe the actual tail behaviour of the loss random variable. In that sense, we could easily construct two loss random variables that would share the same VaR for a given probability p but would have very different tail behaviours, and thus, their actual risks would be completely different even if they would have the same VaR. In fact, VaR simply gives the highest amount that one could expect to lose with a 1-p level of confidence, however, with the remaining 100p %, the probabilities of bigger losses could vary among the different loss functions, and then the actual risk valuation of these financial positions should be different, which is not the case with the VaR method. That´s why it is important when interpreting VaR that the investment positions could in practice give much greater losses than the one VaR states.

In summary, we can see through this statistical definition of VaR that its calculation involves several important factors:

a. The time horizon: the period of time for which the potential loss of the investment or portfolio is estimated. It might be set by a regulatory committee, such as 1 or 10 days.
b. The frequency of the data: the number of times the data value occurs. It might not be the same as the time horizon. Usually, daily observations are used.

c. The significance level 1-p, or probability p: the probability that the losses will result to be greater than VaR. Even if the choice of p is somewhat arbitrary, as big losses occur relatively less frequently in financial markets, losses are often assessed using a small probability, for example 0.05 or 0.01. Hence, the most common significance levels are 1% and 5%.
d. The cumulative distribution function of the loss random variable. This is the focus of econometric modelling since different methods for estimating the CDF give rise to different approaches to VaR calculation.
e. The amount of the financial position or the mark-to-market value of the portfolio and its reference currency.

2.2. Artzner´s (1997) goodness properties of a risk measure estimator

Although several papers define risk in terms of changes in values between two given dates, Artzner et al. argue that, since risk is linked to the variability of the future value of a position, it is more precise to only take into account future values instead. Such variability could come from market changes or more generally to uncertain events. That explains why, while computing VaR of an investment portfolio, the loss of this given financial position for a specific period of time is modelled with a random variable and its associated distribution function.

Furthermore, the problem with risk quantification is that the real distribution function of losses is unknown. Nonetheless, it might be assumed that when we deal with market risk, the state of the world can be illustrated by a set of all prices of all securities, and these data, in contrast, is available. That's why some summary statistics taken out from real data will be employed to adequately estimate the loss distribution function. And, VaR is simply one among all these statistics. In short, a risk measure such as VaR is a mapping from the loss random variable into the real line. This means that it provides an estimate of the potential risk given the real data, even if it does not provide a complete description of the potential losses.

In sum, the goal here should be to select the best possible risk measure, that is, the one that truly describes the losses often encountered in financial markets. In our context, it seems clear that one desirable aspect for a risk measure to be sensible and adequate should be that the estimator must be consistent with the basic theory in finance.

According to Artzner (1997), a coherent risk measure should satisfy four essential conditions. Let η be a risk measure. We say that η is coherent if it satisfies the following four conditions for any two loss random variables X and Y:

1) Subadditivity: $\eta(X + Y) \leq \eta(X) + \eta(Y)$.
2) Monotonicity: If $X \leq Y$ for all possible outcomes, then $\eta(X) \leq \eta(Y)$.
3) Positive homogeneity: For any positive constant c, $\eta(cX) = c\,\eta(X)$.
4) Translation invariance: For any positive constant c, $\eta(X + c) = \eta(X) + c$.

The subadditivity states that the risk measure for a combined position should not be greater than risks of the two positions treated separately. In finance, this is related to diversification. Basically, this concept claims that, by spreading the available resources across different assets and sectors with relatively low correlation, if one area experiences turbulence, the other areas should balance out this effect. Consequently, the risk of a diversified portfolio should not be greater than risks of the individual components. Thus, subadditivity simply expresses the fact that there should be some diversification benefits from combining risks.

The monotonicity simply supports that if one financial position always has greater losses than another position under all circumstances, then its risk measure should always be greater. Clearly, investor's risk perception of a financial instrument that persistently suffers higher volatility or experiences bigger losses than another product will be greater.

The positive homogeneity property has even more important implications. First, it states that doubling a financial position should also double its risk. For instance, the loss of two dollars in any state is as risky as twice the risk of the loss of one dollar. This is the same as saying that the utility of two dollars in one state is worth as much as twice the utility of one dollar in that state. That is, there are no scale effects. Second, this positive homogeneity condition also implies that the incremental risk of the loss of one dollar in any state does not depend on the other states payoffs and so there are no scope effects. Otherwise, it would be profitable to combine assets into particular packages less risky than the parts or to split assets into parts less risky than the whole.

The translation invariance means that there is no additional risk if there is no additional uncertainty because, in statistics, adding a constant to a random variable does not affect its variability. Alternatively, this property can be interpreted as follows: by adding liquidity to a portfolio, its associated risk must be reduced by the same proportion.

Not many risk measure estimators satisfy these goodness properties established in Artzner´s theory. The one in question here, that is, value at risk, is considered to be coherent as long as the random variable loss distribution function is assumed to follow the so-called elliptical distributions such as Normal or Student-T distributions (Franco Arbeláez & Franco Ceballos, 2005). However, without such assumption, VaR is no longer considered as a fully coherent risk measure, because subadditive condition is not met anymore. In fact, value at risk does not behave nicely with respect to addition of risks, thus creating severe aggregation problems and not justifying the diversification concept. Once this is understood, we are now in position to present the different existing methodologies for calculating VaR and so quantifying market risk.

2.3. Different approaches of VaR calculation

Although the VaR concept is not extremely complex, its calculation is not always easy. Initially, three methodologies were developed to calculate VaR of either a single financial asset or an investment portfolio: (i) the Variance-Covariance or Riskmetrics approach, (ii) the Historical Simulation approach and (iii) the Monte Carlo simulation approach. However, these standard models cast doubt on the quality of its estimations since they present considerable shortcomings. As a result, in the last decade, new approaches to VaR calculation have been provided. Broadly, we can nowadays classify these approaches into three different categories.

2.3.1. Non-Parametric Approach

The non-parametric approach seeks to measure VaR of a specific investment or portfolio without making any strong assumption about the returns distribution function. In short, these methods do not require the use of any assumptions about the parameters for the population we are studying here, that is, for the whole financial time series. Hence, the essence of these approaches is to "let data speak for themselves" as much as possible and to use recent returns empirical distributions instead of assumed theoretical distributions in order to estimate VaR (Pilar Abad, Sonia Benito & Carmen López, 2013). Nonetheless, since the idea of all non-parametric approaches is to be able to use the data from the recent past to forecast the risk in the near future we are necessarily assuming in such methods that the near future will be sufficiently similar to the recent past.

This is precisely the main assumption of the most common method inside this category, which is the historical simulation technique (García Dominguez, Meza González & Ventura García, 2017). The core idea here is to use the set of historical prices of a particular portfolio or financial asset in order to obtain scenarios that could be compared with the current position. This application will produce a series of simulated results for the future values of the position, from which we will obtain the VaR. Likewise, new non-parametric approaches have been developed in order to try to utilise better the information available from the financial asset or investment portfolio. For instance, the theory of non-parametric density estimation flourished as a new method that could solve some of the major practical drawbacks of the historical simulation.

2.3.2. Parametric Approach

In contrast with non-parametric estimation methods, the parametric approach applies distributional assumptions and so estimates the parameters of the assumed data distribution. Among this kind of approach, the first model to estimate VaR was presented by JP. Morgan with the name of Riskmetrics methodology. The most important assumption of Riskmetrics is that the continuously compounded daily returns of a portfolio follow a normal distribution. As we will see later on, the major drawbacks of such methodology are precisely related to this strong assumption. As a result, given the deficiencies that this way of estimating VaR has, research has been made in several directions (Abad, Benito & López, 2013).

The most well-known attempt to improve VaR traditional parametric approach involves the use of time series econometric models such as GARCH family models that are more sophisticated volatility models which capture in a better way the specific characteristics observed in financial returns series. Other refined models such as stochastic volatility or realised volatility-based models are also proposed improved procedures for value at risk estimation.

2.3.3. Semi-Parametric Approach

Finally, the semi-parametric approach combines the non-parametric approach with the parametric approach. A bunch of different methods are included in this category, among which the most important ones are the Monte Carlo simulation, the Volatility-weight historical simulation, the Filtered Historical simulation or the approach based on Extreme Value Theory.

Precisely, the Monte Carlo simulation provides an approximation of the portfolio's expected return behaviour using simulations to generate random portfolio's returns based on certain initial assumptions about risk factor volatilities and correlations. Furthermore, the Extreme Value Theory focuses on the evaluation of the probability of occurrence of events or values more extreme than those observed previously in the given sample of a concrete random variable.

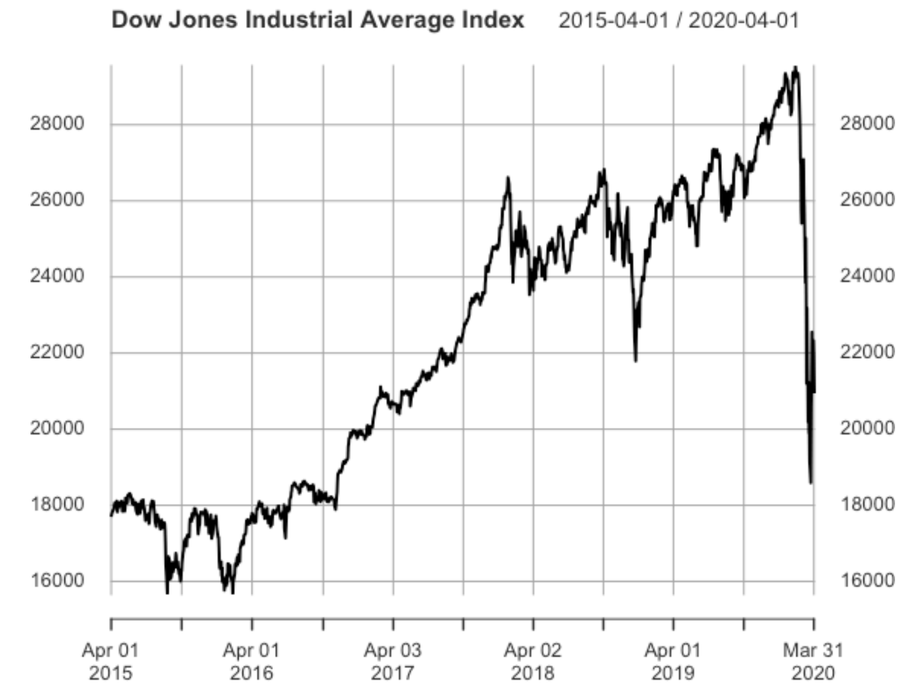
There is no certain evidence of which is the best calculation methodology since it may depend on multiple factors such as the composition of the portfolio or the validity of the starting hypotheses. For this reason, throughout the following sections, we will try to make our small contribution to this field. For simplicity, we will only study in depth five of these forms of calculation: (i) Historical Simulation, (ii) Riskmetrics, (iii) Moving Average Volatility Model, (iv) GARCH family models (under two variants), (v) Extreme Value Theory.

3. STYLIZED FACTS OF FINANCIAL TIME SERIES

Stocks, exchange rates, index prices and other financial time series have some characteristic properties given by the microstructure of the financial markets that distinguish them from other time series. As a matter of fact, to be able to properly quantify market risk and thus calculate value at risk of several financial instruments it is crucial to previously understand the stylized facts of financial time series. The most important particularities will be outlined hereafter through a specific example.

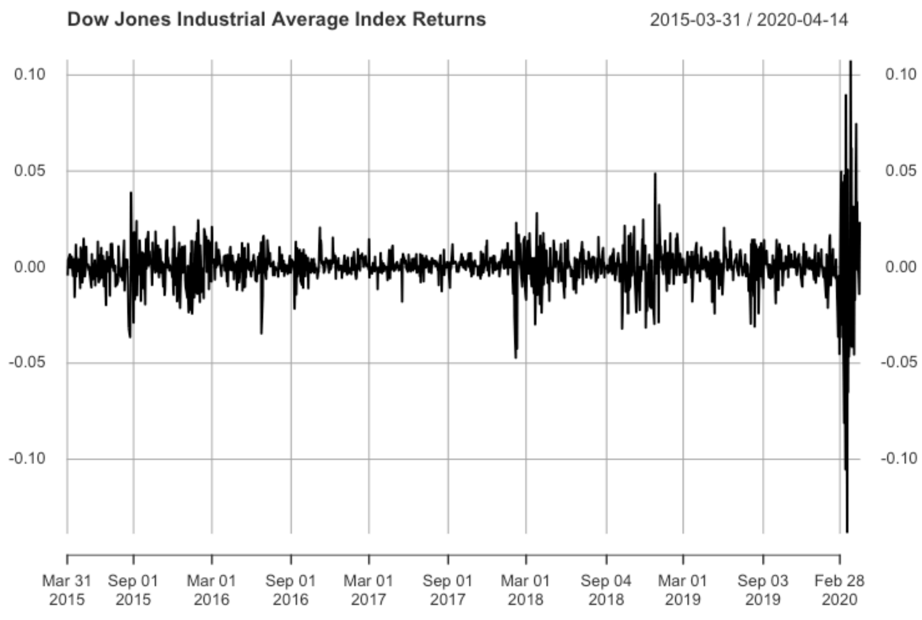
3.1. Non-constant volatility

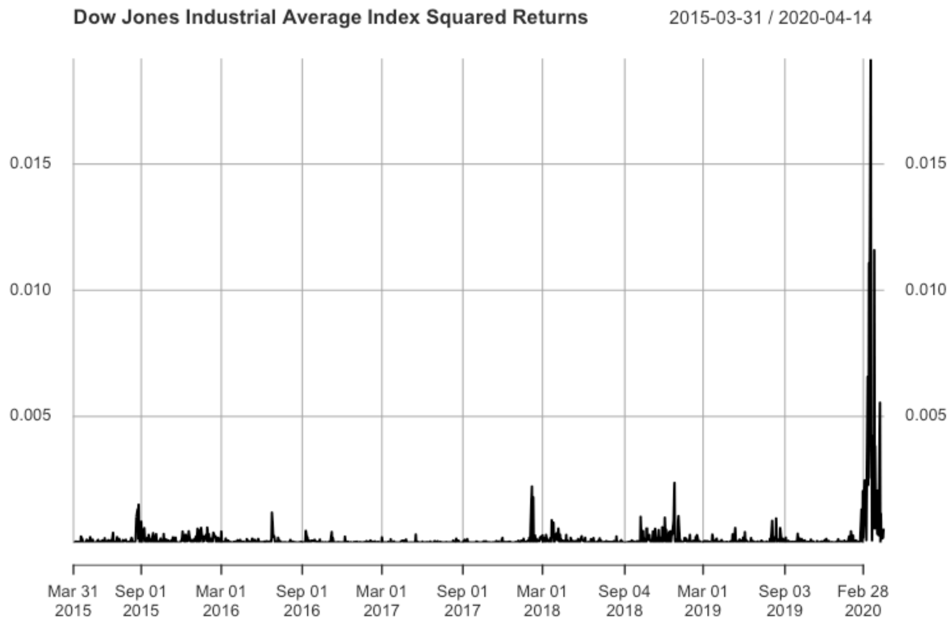
The following picture illustrates the common shape of daily frequency financial time series of the widely followed Dow Jones Industrial Average Index (DJIA). Observe that this time series is non-stationary, that is, its statistical properties such as the mean, variance and autocorrelation are non-constant over time. There is indeed existing empirical evidence that volatility is time varying in financial time series. This fact could be quantitatively proved by performing an Augmented Dickey-Fuller test but this part will be sidestepped here.



3.2. Clustering Volatility

Because of price financial time series non-stationarity, most financial studies involve returns and not prices due to the more attractive statistical properties of returns time series. The following representations show returns and squared returns of DJIA index over the last four years. Squared returns are also plotted because the variance of the series is associated with them, and the variance is the most common measure of dispersion or volatility.





First, note that, in contrast with prices, returns are stationary. Also, check with both figures above that, clearly, periods of high volatility are associated with large squared returns. In addition, we can appreciate the most prominent characteristic of returns, which is the clustering volatility phenomenon. As first noted by Mandelbrot (1963), in financial series "large changes tend to be followed by large changes of either sign, and small changes tend to be followed by small changes". Stated differently, the study of statistical properties of financial time series has revealed that large changes in prices tend to cluster together resulting in persistence of the amplitudes of price changes (A. Kirma & G. Teyssiere, 2005). A quantitative manifestation of this fact is that, while returns themselves are uncorrelated, their squares display a positive, significant and slowly decreasing autocorrelation function. This is also the case for the DJIA index as can be seen in Figures 1 & 2 below.

Figure 1: Correlogram of returns, daily DJIA index returns

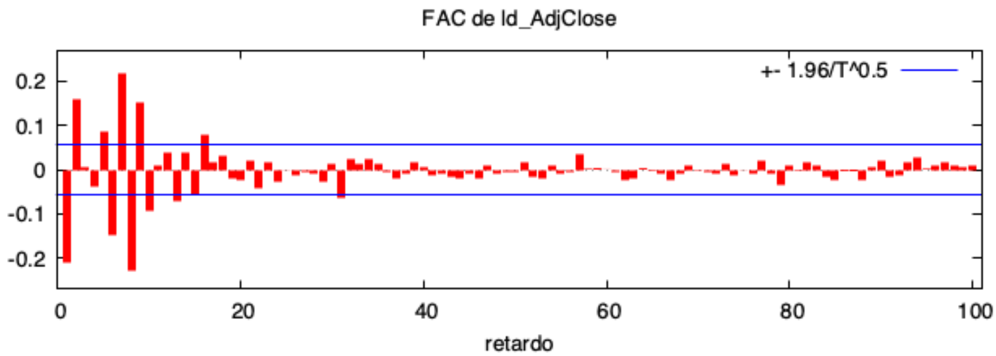
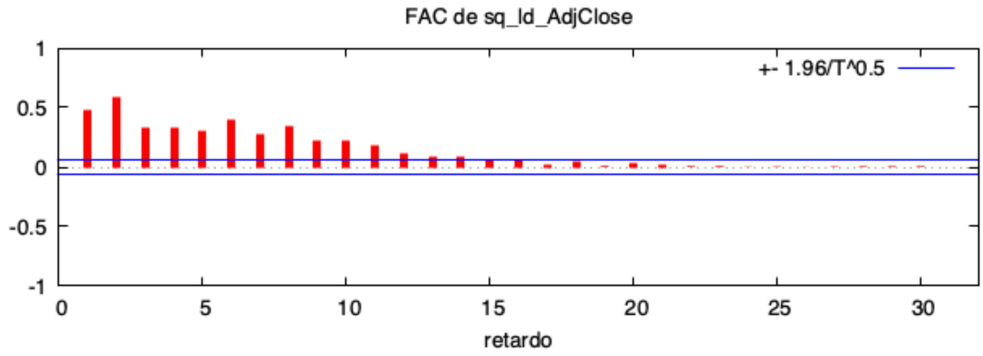


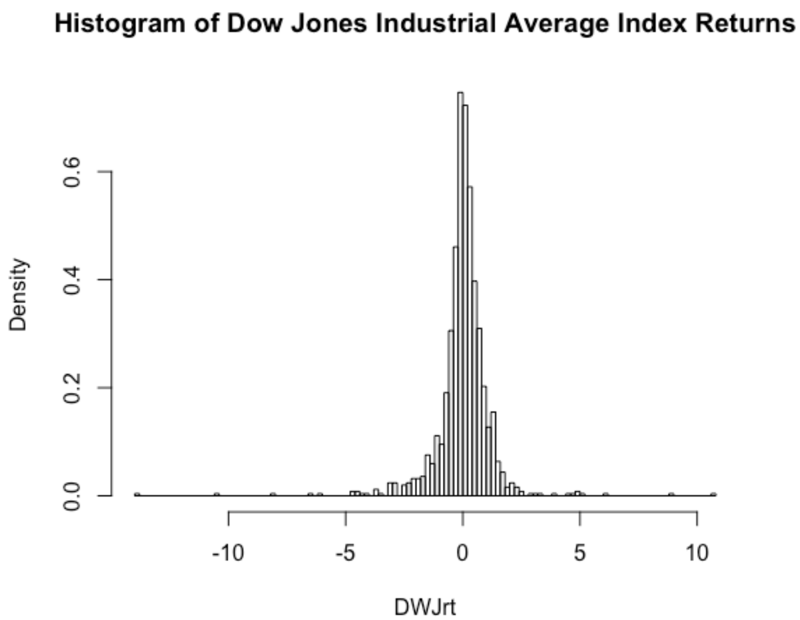
Figure 2: Correlogram of returns squared, daily DJI index returns



We can observe similar behaviour to the one predicted for the Dow Jones Index return series: very small serial correlation in the raw returns but a positive serial correlation in the squared returns. In conclusion, volatility is time varying and also time dependent.

3.3. Leptokurtic distribution

Finally, through DJIA´s histogram, we can guess the last stylized fact of financial time series: the heavy tails of the returns unconditional distribution. These fat tails are captured by the kurtosis, which is much larger than 3. As a consequence, financial time series are often not normally distributed and have leptokurtic distributions. Note that normality could be quantitatively examined using Shapiro-Wilks or Jarque Bera tests.



Once the basic characteristics of financial time series have been understood we are now in position to begin the assessment of different VaR calculation methodologies. For this, we will focus our study on one of the most worldwide followed indexes: the aforementioned Dow Jones Industrial Average. The index is a price-weighted average of 30 blue-chip American stocks, that is, nationally recognized, well-established and financially sound American companies. Since its foundation, the index was designed to serve as a proxy for the broad U.S. economy. The included companies such as Coca-Cola Company, Apple, Microsoft or Walmart are selected depending on its contribution and relevance to their specific sector.

Accordingly, in order to do our research, we have decided to focus on the calculation of the 1-day VaR, since it is the time horizon most commonly used in finance. Furthermore, this daily VaR calculation will be performed over a one-year period to allow us to carry out efficient backtesting techniques. Hence, we will check whether the VaR forecasts have really been met, both quantitatively and qualitatively. Finally, we will manage a three-year time series historical data collected from Yahoo finance. In order to perform our analysis for every single index and VaR method, the R open-source software will be used. The several codes for each VaR calculation methodology will be available in the Appendix section of our work.

4. A PARAMETRIC APPROACH FOR CALCULATING VALUE AT RISK

There are many analytical models describing the fluctuation of financial instruments returns that can be applied to measure market risk. To begin with, through this section, we will discuss several widely used methods included in the parametric approach to VaR calculation. Parametric approaches estimate the risk by fitting probability curves to the data and then inferring the VaR from this fitted curve (Pilar Abad, Sonia Benito & Carmen López, 2013). Hence, these methods rely on distributional assumptions of financial returns such as the Normal or the Student-T distributions.

According to Tsay (2013), for the Normal distribution, if the loss random variable X is normally distributed, say $X \sim N\ (\mu_t, \sigma_t^2)$, then:

$$VaR_{1-p} = \mu_t + z_{1-p}\sigma_t$$

Where z_{1-p} denotes the (1-p) th quantile of the standard normal distribution and the subscript t is used to signify that VaR is time varying. This quantile can be obtained from the normal probability table or from any statistical software package. For instance, $z_{0.95}$ = 1.645 and $z_{0.99}$ = 2.326. Note that in R, we can use the function *qnorm* in order to obtain these two quantiles. Therefore, it can be derived that the core difference between the few parametric models that will be studied here is mainly due to the different ways of estimating the time series mean and volatility parameters.

Similarly, in line with Tsay (2013), for the Student-T distribution, if the loss random variable X satisfies that $Y = (X - \mu_t)/\sigma_t$ is a Student-T distribution with v degrees of freedom, then:

$$VaR_{1-p} = \mu_t + t_{1-p,v}\sigma_t$$

Where is the (1-p) th quantile of a Student-T distribution with v degrees of freedom. For instance, for $v = 5$, we have $t_{0.95}$ = 2.015 and $t_{0.99}$ = 3.365. In R, we can use the command *qt* to obtain these two quantiles.

4.1. Moving Average Volatility Model

The simplest parametric methodology to calculate VaR is the moving average approach. This naïve setting measures the volatility of the time series with a straightforward moving average calculation such that the variance progressively changes with the new information gathered in the last days. Formally, the financial returns variance estimator using the last n days of the historic time series can be expressed as:

$$\hat{\sigma}^2_{t,n} = \sum_{i=0}^{n}(R_{t-i} - \overline{R_n}) / n$$

Where $\hat{\sigma}^2_{t,n}$ and $\overline{R_n}$ correspond to the estimated variance for period t and the returns mean, both calculated from the last n observations. Furthermore, it is well known that daily returns usually exhibit a mean that is not statistically significant (Alonso & Arcos, 2006). Then, $\overline{R_n}$ = 0 and so the expression of value at risk for this particular model is reduced to:

$$Var_{1-p} = z_{1-p}\sigma_t$$

4.2. Exponential Weighted Moving Average Volatility or Riskmetrics Model

The parametric methodology developed by J.P Morgan is also based on the assumption that returns are normally distributed. However, this attempt searched for a more sophisticated volatility model that would capture some of the characteristics observed in the volatility of financial returns. In that respect, Engle (1982) proposed the Autoregressive Conditional Heteroscedasticity

(ARCH) model and Bollerslev (1986) further extended the model by inserting the ARCH generalised model (GARCH). This model specifies two equations; the first depicts the evolution of returns in accordance with past returns, whereas the second patterns the evolving volatility of returns. Hence, this approach takes into account a really important fact of financial time series: the volatility clustering property. Nonetheless, we will see that among all the GARCH family models, the Riskmetrics approach uses one of the simplest ones: the integrated generalized autoregressive heteroscedastic model IGARCH (1,1).

First, for a log return series r_t, let $a_t = r_t - \mu_t$ be the innovation at time t. Riskmetrics sets that the conditional variance σ_t^2 of r_t evolves over time according to the model:

$$a_t = r_t \, , \mu_t = 0$$

$$r_t = \sigma_t \varepsilon_t \, , \{\varepsilon_t\} \sim N(0,1)$$

$$\sigma_t^2 = \alpha + \lambda\sigma_{t-1}^2 + (1-\lambda)r_{t-1}^2 \ \ , 1 > \lambda > 0, \alpha = 0$$

In the particular situation that $\alpha = 0$, this IGARCH (1,1) is equivalent to one of the oldest volatility models: the exponential weighted moving average (EWMA). Therefore, the Riskmetrics method relies on the simple EMWA model. The key feature of such approach is that the impact of past squared shocks on volatility is persistent and so most recent observations are given a higher weight. To see the fact that we are dealing with an exponential smoothing model, we can simply rewrite the previous conditional variance equation and by repeated substitutions we have:

$$\sigma_t^2 = (1-\lambda)\ [x_{t-1}^2 + \lambda x_{t-2}^2 + \lambda^2\sigma_{t-2}^2 + \ldots]$$

which is the well-known exponential smoothing function with λ being the discounting factor. The value of λ is often in the interval [0.9, 1], being 0.94 the typical value recommended by J.P Morgan.

Besides, it can be derived that under the particular IGARCH (1,1) model used in the Riskmetrics approach, the conditional distribution of r_t is $N(0, \sigma_t^2)$. Thus, we only need to deal with volatility forecasts and not mean forecasts for value at risk calculation. Precisely, it turns out that this specific random-walk model has a very good property: the volatility forecasts for all time horizons are easily available (Tsay, 2013). For simplicity, the forecasting method derivations will be avoided.

4.3. A deeper econometric approach: the GARCH (1,1) Model

The research in the framework of the parametric method has moved in several directions. In that sense, many studies have suggested that volatility of returns in stock markets could be effectively modelled and forecasted using the widely known GARCH (1,1) model. Despite many volatility models exist, in our discussion we will focus on this very specific GARH family type as a suitable alternative to the IGARCH (1,1) without drift model used by Riskmetrics. The main objective here will be to examine whether GARCH (1,1) model is a better econometric approach for VaR calculation.

Let r_t be the log return of an asset at time index t. Also, let α_1 be the parameter that measures the extent to which a volatility shock today feeds through into next period´s volatility (Campbell, 1996) and let $\alpha_1 + \beta_1$ be the parameter which measures the rate at which this effect dies over time. Then, in contrast with the previous IGARCH (1,1) model, the conditional distribution of r_t is often assumed to be normal but the mean μ_t and the volatility drift ω are different from zero, and the persistence parameter is lower than one. The GARCH (1,1) model can be written as:

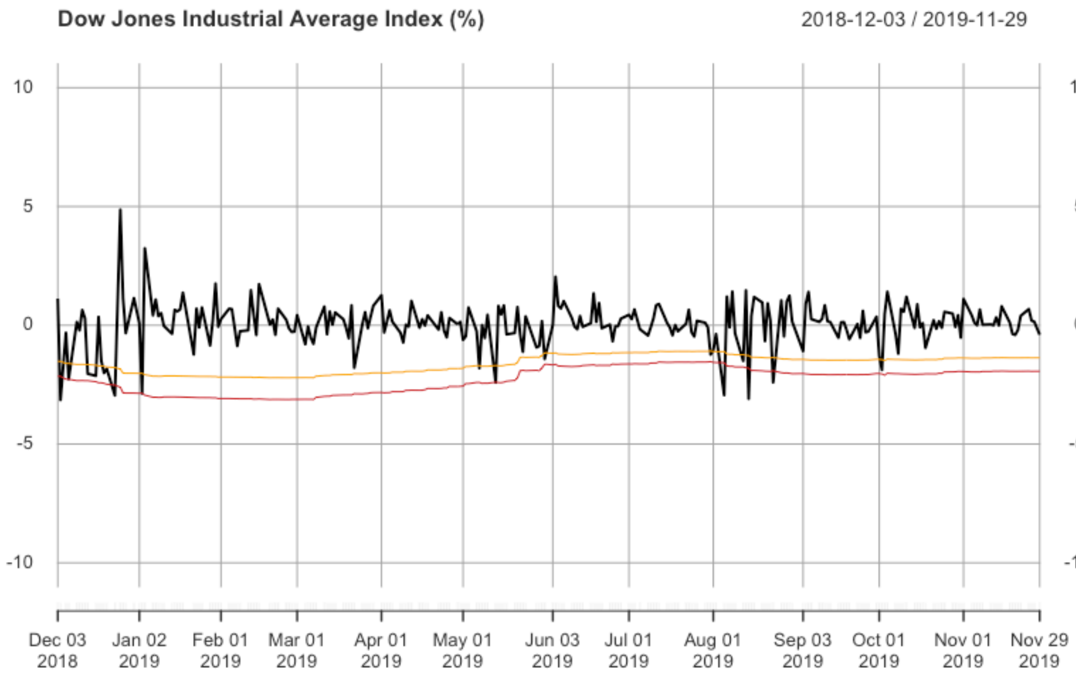
$$r_t = a_t + \mu_t,$$
$$a_t = \sigma_t \varepsilon_t \text{ , } \{\varepsilon_t\} \sim N(0,1)$$
$$\sigma_t^2 = \omega + \alpha_1 \sigma_{t-1}^2 + \beta_1 a_{t-1}^2 \text{ , } \alpha_1 + \beta_1 < 1, \omega \neq 0$$

The parameter estimates of the GARCH (1,1) model are closed to those of IGARCH (1,1) model but, apart from the fact that here μ_t is not directly assumed to be zero, there is another major difference between these two models. In fact, as $\alpha = 0$, the unconditional variance of a_t, and hence that of r_t , is not defined under the IGARCH (1,1) model of Riskmetrics approach. This seems hard to justify for a financial return series (Tsay, 2013).

4.4. Presentation and qualitative discussion of results

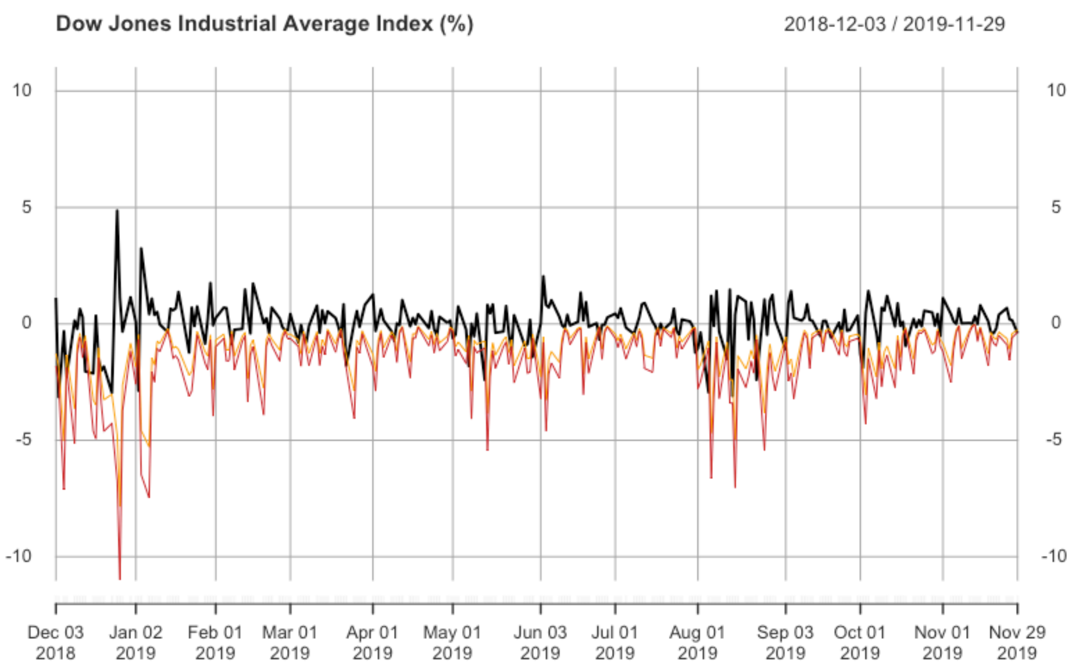
The following graphs show the obtained Dow Jones´s Index value at risk results over a one-year period with the different approaches above explained. Since the Covid-19 pandemic that firstly appeared in the city of Wuhan in December 2019 had dramatic consequences on financial markets, we decided for now to simply focus on the time-period from the 3rd December 2018 to the 29th November 2019. Nonetheless, this high-volatility period caused by the Coronavirus worldwide expansion will be further studied on a later section of this research.

Graph 1: VaR results for the Moving Average Volatility Model



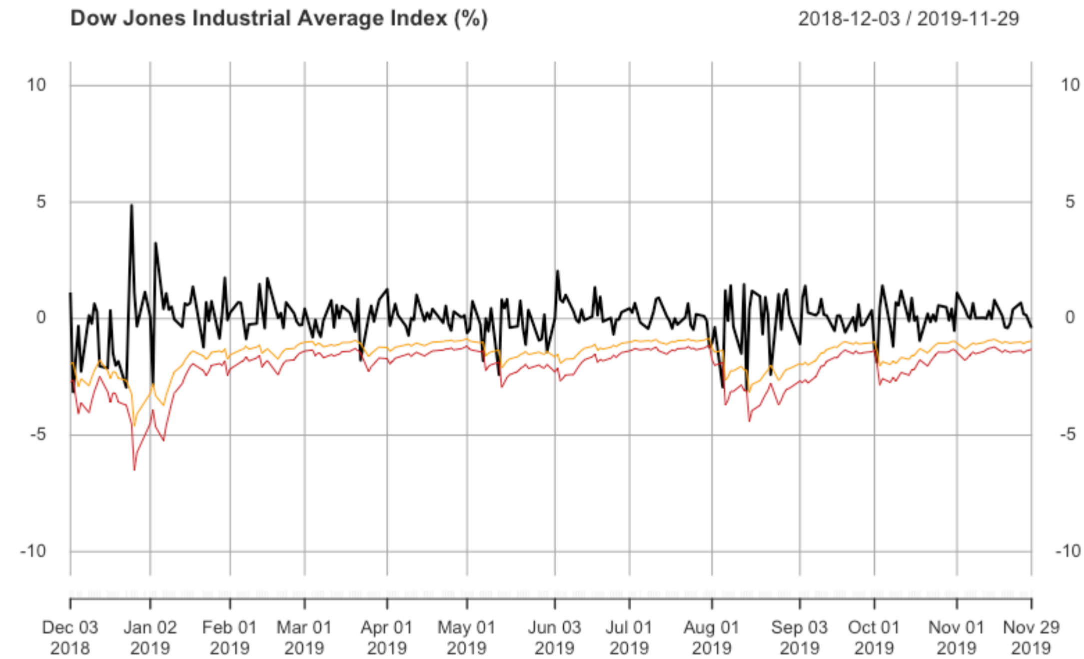
Daily returns appear in black, 95% confidence VaR in orange and 99% confidence VaR in red.

Graph 2: VaR results for the Exponential Weighted Moving Average Volatility Model



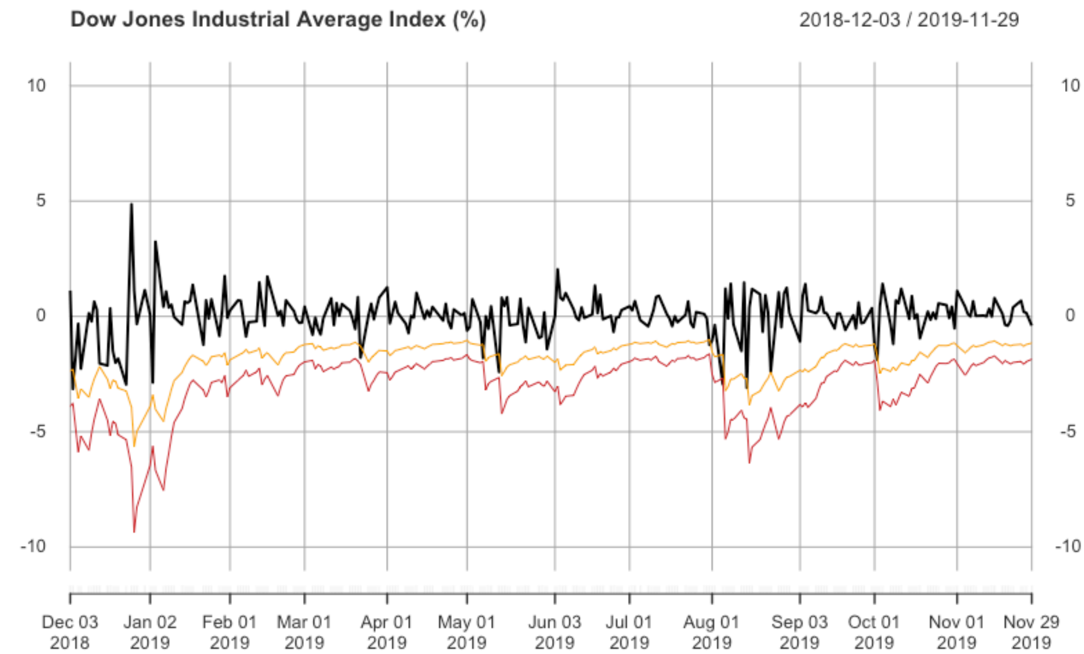
Daily returns appear in black, 95% confidence VaR in orange and 99% confidence VaR in red.

Graph 3: VaR results for the GARCH (1,1) Model with Normal distribution assumption



Daily returns appear in black, 95% confidence VaR in orange and 99% confidence VaR in red.

Graph 4: VaR results for the GARCH (1,1) Model with Student-T distribution assumption



Daily returns appear in black, 95% confidence VaR in orange and 99% confidence VaR in red.

Qualitatively, significant differences can be noticed in the results obtained for each VaR calculation methodology. In fact, while the moving average volatility model (MA) reflects a much more constant line over time, the other two approaches seem to be more adapted to the reality of the market. In fact, GARCH (1,1) models and specially the Riskmetrics approach are more sensible to changes in volatility of returns than the MA volatility model is. Furthermore, looking at the graphs above, the GARCH (1,1) models seem to have performed particularly well, specially the one with the Student-T distribution assumption. Finally, remember that VaR key goal is to estimate the maximum potential loss in a given time horizon for a specific level of confidence, and so the fact that a model seems to be more accurate to estimate the markets future returns doesn't necessarily imply that it is a better VaR methodology. For instance, it could be the case that the MA almost constant line could have had a better performance in what respects the main goal of value at risk. A quantitative analysis is required to check all these suggestions. In that respect, section 7 of this work contains performance results using backtesting techniques for every studied VaR methodology, also including non-parametric and semi-parametric methodologies.

4.5. Major drawback of the studied parametric methods

The biggest weakness of the studied VaR parametric methodologies is the assumption of normality of returns. In fact, as we saw in previous sections, financial data time series are characterized for having leptokurtic distribution of returns, meaning that the skewness coefficient is in most cases negative and statistically significant. This result is not in accordance with the properties of a normal distribution, which is, by definition, symmetric. In addition, the empirical distribution of financial returns has been documented to exhibit a significantly excessive kurtosis. As a consequence, the strong assumption of normality is not fulfilled in practice. A possible alternative would be to assume a Student-t distribution of results instead of a Normal distribution. This is precisely what we did in the second GARCH (1,1) model estimation, where we assumed a 5 degrees of freedom Student-T distribution as recommended by Tsay (2013).

Besides, taking this drawback into account, one could move to a framework of non-parametric methods which does not require any assumption of the distribution of returns. Accordingly, the study of non-parametric approaches is the main purpose of the following paragraphs.

5. A NON-PARAMETRIC APPROACH FOR CALCULATING VALUE AT RISK

In contrast with parametric methods, which depend on strong theoretical assumptions of the underlying properties of the data, non-parametric approaches can accommodate wide tails, skewness or any other non-normal features in financial observations (Dowd, 2002). In that sense, the historical simulation VaR methodology studied here below depends solely on historical data.

5.1. Historical Simulation VaR

The historical simulation method is the simplest approach to risk prediction that uses the empirical distribution of past returns to generate VaR. Despite no underlying assumptions of normality are needed, the approach is based on the assumption of history repeating itself. That is, the results are completely dependent on the data set and so it is implicitly assumed that past returns are a good and complete indicator of the expected future returns. This trade-off is at the heart of the historical simulation debate. Indeed, we will here not consider any sophisticated bootstrapping technique, such as the block bootstrap historical simulation. This method consists in constructing subsamples of specific size where the returns have the same probability of being selected. Then, the value at risk of each of these blocks or subsamples is calculated and finally an average of the several obtained VaR results is used as a final estimate. However, for simplicity, we will here apply the sample method, which solely takes the results of the chosen sample as a reference and projects them into the future.

To run such sample historical simulation, we need to derive the returns time series of the chosen financial instrument for the selected period, just as it would have been done in the Riskmetrics model or any other parametric method. However, the difference here is that the data are not used to estimate variance and covariance, since the changes in the portfolio or the asset over time give us all the information we need to compute VaR. In fact, once the returns have been calculated, we simply make the assumption that these returns may occur tomorrow with the same likelihood, and so, a new price time series is created taking the last available price as the reference value and simulating a whole new set of prices based on the past returns. The idea is basically to consider that tomorrow´s price would be any among all these simulated possibilities, which take into account what would have happened if each of the previous past returns had occurred.

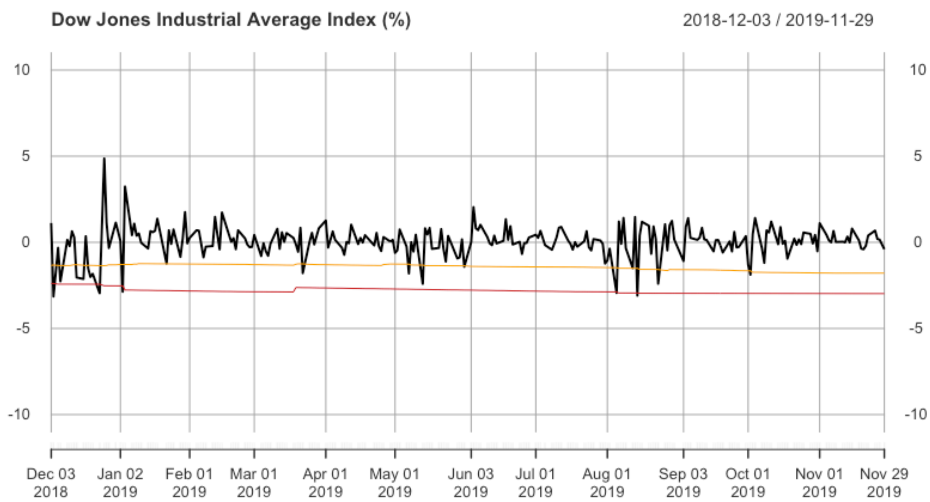
Finally, simulated profits and losses can be computed for each specific scenario. It should be noted, therefore and according to the used simulation method, that the predicted loss cannot be greater than that of the historical loss (Tsay, 2013). Once these results are obtained for each day of the observation period, a distribution of expected returns is generated. As a result, the 95 or 99 percentiles of such distribution directly are the value at risk results for 95% and 99% levels of confidence respectively.

While historical simulations are popular and relatively easy to run, they do come with baggage. The main disadvantage of the historical simulation method is given by the characteristics of the used data, which assumes that no event that has not occurred in the past can occur in the future. As a consequence, if the data period is unusually quiet, historical simulation will often underestimate risk, whereas if we are dealing with a particularly volatile, period historical simulation will often overestimate it. Besides, a related argument can be made about the way value at risk is computed through historical simulation. In fact, all data points are weighted equally and so the price changes from trading days three years ago affect VaR in the exactly same proportion as price changes from last two weeks do. Ultimately, another problem arises when new assets from emerging market, for which no historical information is available, are introduced to the analysis.

5.2. Presentation and qualitative discussion of results

The following were the 1-day value at risk results obtained for the Dow Jones index from December 2018 to December 2019.

Graph 5: VaR results for the Historical Simulation Model



Daily returns appear in black, 95% confidence VaR in orange and 99% confidence VaR in red.

Looking at this graph we can see that historical simulation VaR remains fairly constant over time and hardly varies when sudden changes in profitability occurred. Furthermore, we also appreciate significant differences depending on the value at risk level of confidence. In that respect, results suggest that, whereas 95% confidence VaR seems to have a poor efficiency rate, the 99% confidence VaR seems an acceptable measure, despite the maximum losses do sometimes exceed the VaR predictions. Backtesting tests will allow us to later corroborate these findings.

6. A SEMI-PARAMETRIC APPROACH FOR CALCULATING VALUE AT RISK

It should be clear by now that normal distributions are not commonly able to reflect financial markets real behaviour. The assumption made about the distribution function of returns is key to some already studied methods such as the Riskmetrics or the GARCH models. As an alternative, a new approach to value at risk has recently been proposed: The Extreme Value Theory (EVT). The goal of EVT is to solve the problem of heavy tails of financial data distributions by focusing on the quantification of the probabilistic behaviour of unusually large losses (McNeil & Frey, 2000).

6.1. Extreme Value Theory

Theoretically, value at risk tries to capture the maximum loss under normal market situations. However, extreme events such as the 2008 major financial crisis really cast doubt on the VaR ability to accurately measure risk in worst case scenarios. Therefore, the goal of the Extreme Value Theory approach to value at risk is to solve such problem and consider the possibility of occurrence of extreme events. In that sense, EVT aims to calculate the possible losses under sever market situations (Claro, Contador & Quiroga, 2006).

According to the EVT, the precise form of the yield distribution tails is known and can be modelled. More importantly, the tail distribution is independent from the yield probability distribution. Therefore, according to this theory, a single probability law can efficiently describe the tails behaviour of almost all probability distributions. The Extreme Value Theorem provides us with the possible form that tails distribution could take. Assume that losses x_t are serially independent with a common cumulative distribution function $F(x)$. Then, according to the EVT, there is sufficient evidence that the limiting distribution $F_*\ (x)$ of the cumulative distribution function $F(x)$ of x_t necessarily takes one of the following forms (Gnedenko, 1943):

- Gumbel Distribution - Type I: $\varepsilon = 0$

$$F_*(x) = e^{-e^{-x}}, x \in \mathbb{R}$$

- Fréchet Distribution - Type II: $\varepsilon > 0$

$$F_*(x) = e^{-(1+\varepsilon x)^{-\frac{1}{\varepsilon}}}, x > -\frac{1}{\varepsilon}$$

$$F_*(x) = 0, x \leq -\frac{1}{\varepsilon}$$

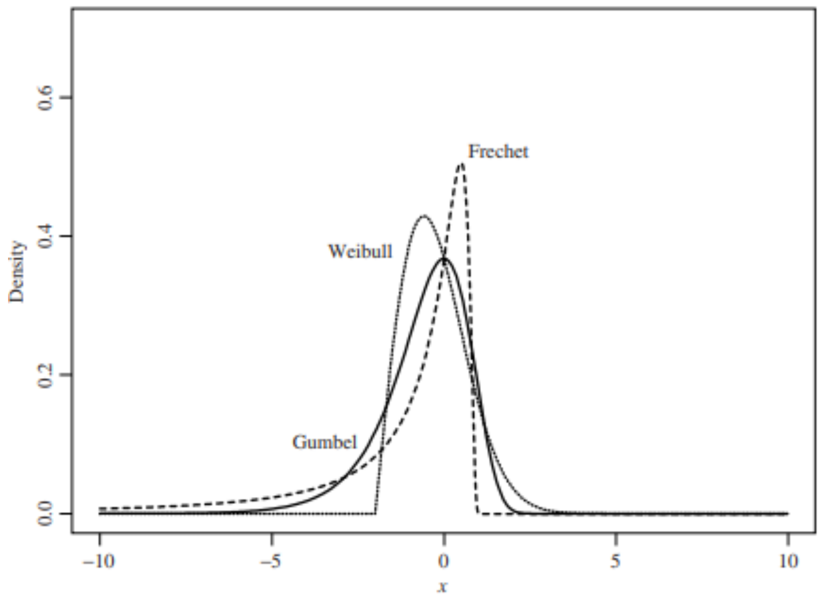
- Weibull Distribution - Type III: $\varepsilon < 0$

$$F_*(x) = e^{-(1+\varepsilon x)^{-\frac{1}{\varepsilon}}}, x > -\frac{1}{\varepsilon}$$

$$F_*(x) = 1, x \leq -\frac{1}{\varepsilon}$$

Where the parameter ε is referred to as the shape parameter that governs the tail behaviour of the limit distribution function. Figure 1 below represents graphically each particular distribution. We can observe that the right tail of the distribution declines exponentially for the Gumbel family, by a power function for the Fréchet family, and is finite for the Weibull family (Tsay, 2013). Besides, the Fréchet distribution converges more slowly than the Weibull distribution, since in the tails it decreases according to the power of the variable values.

Figure 1: Extreme Value Theory limit distribution functions.



Source: An Introduction to Analysis of Financial Data with R (Tsay, 2013)

Later on, Jenkinson (1955) combined the Gumbel, Fréchet and Weibull distributions presented by Gnedenko and proposed a single formula for the limit distribution of a set of independent and identically distributed random values representing observations. This extreme value distribution is referred to as the generalized extreme value distribution (GEV), and its cumulative distribution function has the following form:

$$H(x) = \begin{cases} e^{-(1+\varepsilon x)^{-1/\varepsilon}}, & if\ \varepsilon \neq 0 \\ e^{-e^{-x}}, & if\ \varepsilon = 0 \end{cases}$$

Therefore, the Extreme Value Theory sets that the tail behaviour of the cumulative distribution function $F(x)$ of the losses x_t of a particular financial instrument determines the limiting distribution $H(x)$ of the cumulative distribution function $F(x)$ of x_t. As we said, $H(x)$ has three types depending on the shape parameter ε, but since in finance we usually find heavy tails (ξ > 0), while applying EVT to calculating VaR we will be mainly interested in the Fréchet family that includes stable and Student-T distributions. Then, in the approach to VaR calculation using the Extreme Value Theory, the shape parameter that will be estimated is the one of the Fréchet distribution, which is a special case of the GEV. Besides, we will also need to derive the usual location μ and scale σ parameters of the GEV.

Consequently, for a given small upper tail probability p and a length n of subperiods, the VaR of a financial position with loss variable x_t is:

$$VaR = \mu_n - \frac{\sigma_n}{\varepsilon_n}\{1 - [-n \ln(1-p)]^{-\varepsilon_n})\} \text{ with } \varepsilon_n > 0$$

Nevertheless, before applying EVT to assessing risk and computing VaR, we need to estimate the unknown parameters of the generalized extreme value distribution: that is, the shape parameter ξ, the location parameter μ and the scale parameter σ. These parameters can be estimated by using either parametric or nonparametric methods (Tsay, 2013).

Among the parametric approach, the traditionally used models are the Block Maxima Model (McNeil, 1998), and the Maximum Likelihood Method. The Block Maxima approach involves dividing the historic dataset into blocks or subsamples of n observations each, and looking for the maximum of all the subsample maxima. Then, the maximum values of each block are collected and these selected observations form the so-called maxima blocks (Abad, Benito & López, 2013), from which the unknown parameters of the extreme value distribution are derived. Clearly, the estimates obtained may depend on the choice of subperiod length n (Tsay, 2013). Besides, similar to this approach, the Maximum Likelihood Method states that, under the independence assumption, the likelihood function of the subperiod maxima can be easily obtained and used to obtain maximum likelihood estimates of ξ, μ and σ.

Finally, within the nonparametric approach, the shape parameter ξ can be estimated with two different methods proposed by Hill (1975) and Pickands (1975), and that are referred to as the Hill estimator and the Pickands estimator, respectively (Tsay, 2013). There is no need here to consider subsamples as there was in the parametric approach.

Here below are the results for the shape parameter ξ for the negative daily log returns of Dow Jones Index from November 2015 to November 2019 calculated with two different methodologies: The Hill Estimator and the Maximum Likelihood Model. As we can observe, in both cases the shape parameter is statistically significant from zero and positive for a 95% confidence level. Therefore, the normality assumption of the tail behaviour of the cumulative distribution function of the losses of the Dow Jones Index can be rejected. In fact, the obtained results suggest that, as was expected, the tail behaviour of this cumulative distribution can be described by a Fréchet distribution.

Table 1. Results of the Hill Estimator for Negative Daily Log Returns of Dow Jones Index from November 2015 to November 2019.

q	**110**	**130**	**150**
Shape parameter $\boldsymbol{\varepsilon}$	0.6460	0.6682	0.7545
Standard Errors	(0.0615)	(0.0586)	(0.0616)

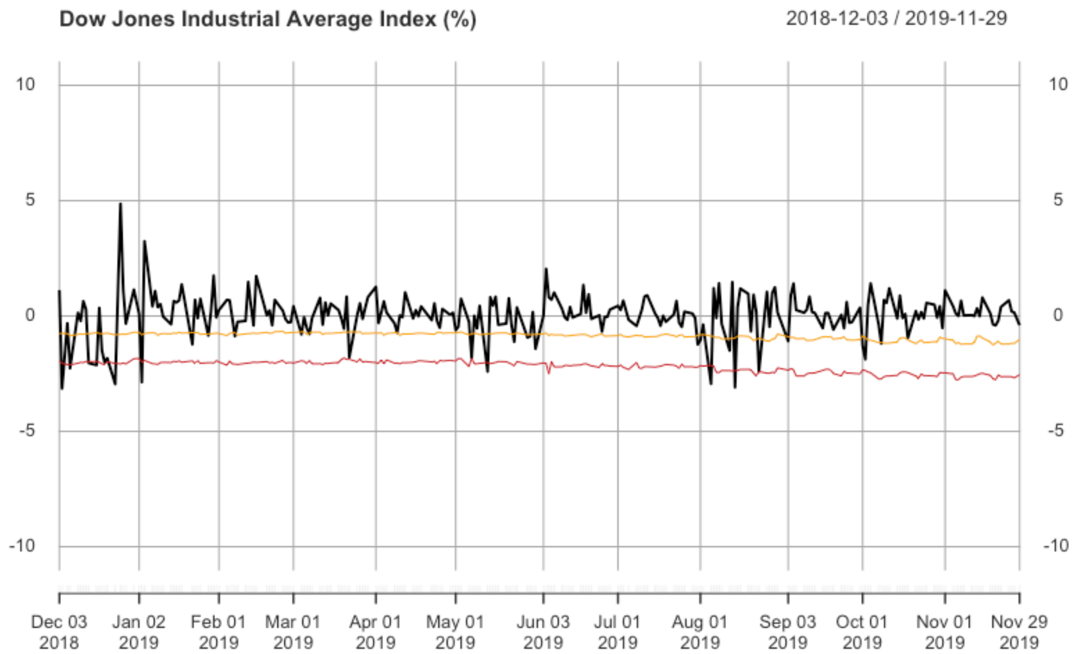
Table 2. Maximum Likelihood Estimates for the Extreme Value Distribution for Negative Daily Log Returns of Dow Jones Index from November 2015 to November 2019, n = 21.

Parameters	**Shape $\boldsymbol{\varepsilon}$**	**Scale $\boldsymbol{\sigma}$**	**Location $\boldsymbol{\mu}$**
Estimation	0.5697	0.4827	0.8744
Standard Errors	(0.2106)	(0.0874)	(0.0884)

6.2 Presentation and qualitative discussion of results

As with the other methods, we calculate the Dow Jones´s index VaR through the Extreme Value Theory approach over a one financial year period. It is important to highlight that we considered here a length n of the subsamples equal to 21, which corresponds approximately to a financial month. The graph obtained looks as follows:

Graph 6: Extreme Value Theory Model



Daily returns appear in black, 95% confidence VaR in orange and 99% confidence VaR in red.

As we can observe, the EVT VaR is not an almost strictly perfect line as in the case of the historical simulation approach. Nonetheless, this method neither seems extremely sensitive to big profitability changes as occurred in the Riskmetrics or GARCH (1,1) models. In overall, EVT value at risk results could be described as a set of values that oscillate over a relatively constant value. Furthermore, contrary to what we expected, this relatively new approach does not seem to improve the previous existing ones, as we can graphically appreciate that the real losses often exceed the VaR.

The previous qualitative analysis of the obtained results for each respective VaR methodology only allowed us to come up with simplistic judgements that definitely need to be considered quantitatively. In that respect, a more thorough analysis will be done in the following section by performing several backtesting techniques. This final step will finally lead us to the main purpose of this research, which is to evaluate the effectiveness of each particular VaR calculation methodology. Furthermore, we will carry out an interesting study on the recent impact of Covid-19 worldwide virus expansion to appraise the adjustment ability of the different VaR approaches to high levels of risk and volatility.

7. BACKTESTING: WHICH IS THE BEST VAR CALCULATION METHOD?

We are now in position to study in a quantitative form if the estimates made by the different VaR models were fulfilled in the real scenario using a Backtesting process which will allow us to validate the accuracy and precision of the different approaches. In that respect, Backtesting is a set of statistical procedures designed to check if the real losses are in line with VaR forecasts or not (Jorion, 2001). It is based on the comparison of the observed P&L to these VaR forecasts. Ultimately, the comparison of realized and expected losses clarifies whether risk was overestimated or underestimated.

The most common test of VaR model is to count the number of days or holding periods of other length where portfolio losses exceed VaR estimates. It therefore comes down to statistical analysis to determine whether this number is reasonable or not, and thus whether the model will be accepted or rejected. These backtesting models that focus only on counting the number of exceedances are called unconditional coverage tests, and will be the ones taken into account here.

Denoting the number of exceedances as x and the total number of observations as T, we may define the following rate $p^* = x/T$ which describes the real proportion of times where the actual loss was higher than the estimated VaR. In an ideal case, this rate would reflect the selected confidence level. This optimal situation could be statistically described as follows. For a given confidence level (1-p) %, we set the null hypothesis that the frequency of tail losses is equal to p, that is, $p = p^*$. Hence, assuming that the model is accurate, the observed rate x/T should act as an unbiased measure of p and thus converge to the significance level as sample size is increased (Jorion, 2001). The goal is basically to contrast whether the real proportion of exceedances is significantly different from p or not.

Since it could be considered that each outcome in the dataset either produces a VaR violation or not, this sequence of results could be described as a successes and failures sequence, which is clearly a Bernoulli process. This implies that the number of exceedances x could be treated as a realization of a binomial random variable X which takes a value of either 0 or 1 and that follows a binomial probability distribution:

$$f(x) = \binom{T}{x} p^x (1-p)^{T-x}$$

Finally, using the Central Limit Theorem, as the number of observations increase, the binomial distribution can be approximated to a normal distribution such that:

$$z = \frac{x - pT}{\sqrt{p(1-p)T}} \approx N(0,1)$$

By utilizing this binomial distribution, we will be able to examine the accuracy of the respective VaR models. However, as with any other statistical hypothesis testing, when conducting a backtesting technique that either accepts or rejects a null hypothesis of the model being efficient, there is a trade-off between two types of errors. On the one hand, a type 1 error refers to the rejection of a true null hypothesis, which will imply rejecting a correct model. On the other hand, a type 2 error is the non-rejection of a false null hypothesis, that is, the possibility of accepting an incorrect model. A statistically powerful test would efficiently minimize both of these probabilities (Jorion, 2001). Here below are presented the exceptions results obtained for each of the VaR models. We will now proceed to quantitatively analyse them by using two different conditional coverage tests: The Kupiec´s test and the Basel Committee´s Traffic Light Coverage test. We will compare the conclusions to be drawn for each.

	Number of exceedances	
	95% Confidence Level	**99% Confidence Level**
Moving Average	17	7
EWMA or Riskmetrics	38	29
GARCH (1,1) Normal Distribution Assumption	14	6
GARCH (1,1) Student-T Distribution Assumption	9	1
Historical Simulation	17	5
Extreme Value Theory	32	9

7.1 Unconditional Coverage backtesting

7.1.1. Kupiec Test

The most widely known backtesting technique was suggested by Kupiec in 1995. Kupiec´s "proportion of failures" (POF) coverage test measures whether the number of exceedances is consistent with the confidence level. The idea is to examine whether the observed "failure rate" p^* is significantly different from the "failure rate" p suggested by the confidence level. In that sense, under the null hypothesis of the model being accurate, the number of exceedances should follow the binomial distribution discussed previously. Hence, the null hypothesis for the POF test is:

$$H_0 : p = p^* = x/T$$

Furthermore, according to Kupiec, the POF test is best conducted as a likelihood-ratio (LR) test. In fact, rather than directly calculating the probabilities from the binominal distribution of the random variable X of the number of exceedances, the POF tests uses this distribution to construct a likelihood ratio which turns to be the test statistic asymptotically distributed as a one degree of freedom chi-squared χ^2. It takes the following form:

$$LR_{POF} = -2ln\left(\frac{(1-p)^{T-x}p^x}{\left[1-\left(\frac{x}{T}\right)\right]^{T-x}\left(\frac{x}{T}\right)^x}\right) \approx \chi^2(1)$$

If the value of the LR_{POF} exceed the critical value of the $\chi^2(1)$ distribution, the null hypothesis will be reject and consequently the model will be considered as inaccurate. Here after are the results obtained for each model.

Kupiec Test Results:

95% Confidence Level	
Expected Exceedances	12.5
Critical Value	3.8415

Methodology	Observed Exceedances	LR	Conclusion
Moving Average	17	1.5403	Do not Reject H_0
EWMA	38	36.3427	Reject H_0
Garch (1,1) Normal	14	0.1827	Do not Reject H_0
Garch (1,1) Student-T	9	1.1383	Do not Reject H_0
Historical Simulation	17	1.5403	Do not Reject H_0
Extreme Value Theory	32	22.8072	Reject H_0

99% Confidence Level	
Expected Exceedances	2.5
Critical Value	6.6349

Methodology	Observed Exceedances	LR	Conclusion
Moving Average	7	5.4970	Do not Reject H_0
EWMA	29	92.1027	Reject H_0
Garch (1,1) Normal	6	3.5554	Do not Reject H_0
Garch (1,1) Student-T	1	1.1765	Do not Reject H_0
Historical Simulation	5	1.9568	Do not Reject H_0
Extreme Value Theory	9	10.2290	Reject H_0

Analysing the tables above we discern that both the EWMA or Riskmetrics model and the Extreme Value Theory model are not accurate for measuring value at risk, or at least haven´t done it properly over the last year for the Dow Jones index. In fact, the null hypothesis that the frequency of tail losses is equal to the significance level is rejected in both cases for 95% and 99% confidence levels. As a matter of fact, with 29 exceptions, the Riskmetrics model has clearly underestimated risk over the last year and seems to be an outdated model.

Furthermore, the other three value at risk models seem to be better fitted according to the Kupiec´s test. In fact, the null hypothesis cannot be rejected for either 95% or 99% confidence levels. Remarkably, for a 95% level of confidence, the GARCH (1,1) under the Normal distribution of returns assumption has performed the best, whereas for a 99% confidence level, the model with the better outcomes is the GARCH (1,1) under the Student-T distributional assumption. Hence, in consonance with our results and the Kupiec test, the GARCH (1,1), the Historical Simulation and, surprisingly, the Moving Average, have achieved and acceptable level of exactitude for the Dow Jones index over the last financial year.

However, certainly the GARCH (1,1) methodology, especially the one with the associated Student-T, is the one with the highest merit and the best outcomes, since it has been able to rigorously estimate the shape of the future returns as we saw graphically in previous sections. The good performance of the GARCH (1,1) model under the Normal distribution assumption should also be noted, despite this method seems to suffer for a 99% confidence level. Besides, the other two approaches, i.e. the Historical Simulation and Moving Average, looked like a fairly constant line which almost never reacted to sudden volatility changes over the financial year as the GARCH (1,1) model did. On the report of the Kupiec´s test results, in both cases this line has been proven to efficiently estimate the one-day maximum potential loss of the Dow Jones index but the VaR results are less sophisticated and specific and so, in short, less representative of what really happened in financial markets. In conclusion, it could be considered that three methods achieved the main goal of VaR, which is to precisely estimate the maximum potential loss of the index, but the GARCH (1,1) model added value needs to be pointed out.

Nevertheless, the biggest shortcoming of Kupiec´s POF tests is that the test is statistically weak with sample sizes that are consistent with the current regulatory framework of one year. Thus, our model backtesting should not rely solely on Kupiec´s test. An alternative within conditional coverage tests is the regulatory backtesting process.

7.1.2. Basel´s Committee regulatory framework

The 1996 Amendment to the Basel Accord imposed a capital charge on banks for controlling market risk. Since then, many financial institutions developed their own independent internal risk management functions to assess their capital requirements. However, regulators needed to be satisfied that the value at risk measures were sound, and so backtesting processes had to be carried out necessarily. In that respect, the Basel Committee specified a methodology for backtesting the one-day value at risk against daily profits and losses. This technique, also known as “traffic light” backtest, is based on the consideration that the occurrence of an exception on a given day is independent of the outcome of any other day (Balzarotti, Del Canto & Delfiner, 2001).

Basel committee imposes a 1% significance level. Similar to the previous Kupiec test, if we work with a 99% confidence level, the probability of having X exceptions in a period n of 250 days corresponding to a financial year is binomially distributed. Then, based on the number of exceedances experienced during the period, the value at risk measure is categorized into three coloured zones. First, the green zone corresponds to backtesting results that do not suggest problems with the quality of the internal model. Second, the yellow area opens up questions about the model but does not allow to concrete into a definitive conclusion. Third, the red zone does indicate a clear problem with the model.

The limits of the three zones have been established for the purpose of adjusting the two types of statistical errors, namely, the possibility of mistakenly rejecting a suitable model (type I error) and the possibility of accepting an unsuitable model (type II error). Hence, a cut-off level can be determined in the number of exceptions from which we will reject the hypothesis that the model is adequate, with a level of confidence that can be obtained according to the binomial cumulative probability distribution function. In Basel, the cut-offs have been determined as follows: the yellow zone starts where the cumulative probability exceeds 95% and the red zone where it exceeds 99.99% (Balzarotti, Del Canto & Delfiner, 2001)

Accordingly, the zones are defined in such a way that the green zone brings together cases which do not suggest the possibility of accepting an inappropriate model. The red zone is one in which a type I error is remote and at the same time it is extremely unlikely to make a type II error. Between these two zones, however, there is a yellow zone in which the results could be consistent with suitable models or not (Basel Committee on Banking Supervision, 1996).

Zone	Number of exceedances	Cumulative probability assuming q*= 0.99
	0	0.0811
	1	0.2858
Green	2	0.5432
	3	0.7581
	4	0.8922
	5	0.9588
	6	0.9863
Yellow	7	0.9960
	8	0.9989
	9	0.9997
Red	10 or more	0.9999

Source: Basel Committee on Banking Supervision, 1996

If we apply these requirements above to our table of exceptions previously presented, we see that we can only one of the analysed models could be classified within the green zone: the GARCH (1,1) model under a Student-T distribution. The remaining studied models would be located within the yellow zone, except for the Riskmetrics one, which, with 29 exceptions, would be in the red zone. Within this yellow zone, the model with the best results has been offered by the historical simulation model that only accounts 5 exceptions and is very close to the green zone. Then, the GARCH (1,1) under a Normal distribution assumption, the Moving Average, and the Extreme Value Theory account for 6, 7 and 9 exceptions respectively and so the probability of accepting imprecise models increases in that same order.

All this analysis allows us to make a comparison between the different approaches as well as to determine which ones seem more suitable according to the data we have used**.** Firstly, both backtesting methodologies confirm that the Riskmetrics model seems to be the one that has predicted the maximum loss events the worst, and therefore presents a performance that discourages its use. Similarly, the Extreme Value Theory approach also had a very poor performance, which is something very unexpected since recent empirical literature on VaR methodology claims good percentages of success of the EVT.

Secondly, although for historical simulation or moving averages methods we cannot reject the null hypothesis that the frequency of tail losses is equal to the significance level according to the Kupiec test results, we cannot roundly state either that their performance has been satisfactory, since the number of exceptions they present is still greater than expected and desired. In short, only the GARCH (1,1) model, and specially the approach under the Student-T distribution of returns seems a sufficiently good enough way to estimate value at risk according to our results on the studied period for the Dow Jones Index. In that sense, this relatively new proposal within parametric methodologies to estimate VaR has clearly outperformed the traditional Riskmetrics one and also the already existing non-parametric approaches.

Now that the performance of each methodology has been judged in a relatively stable period, it would be interesting to examine which model best reacted under a different and extreme situation with high volatility rates, and see whether the results obtained are especially different to the ones already exposed or not. For this purpose, the Covid-19 pandemic will be introduced in our analysis and so the data from December 2019 to April 2020 ignored until now will be studied.

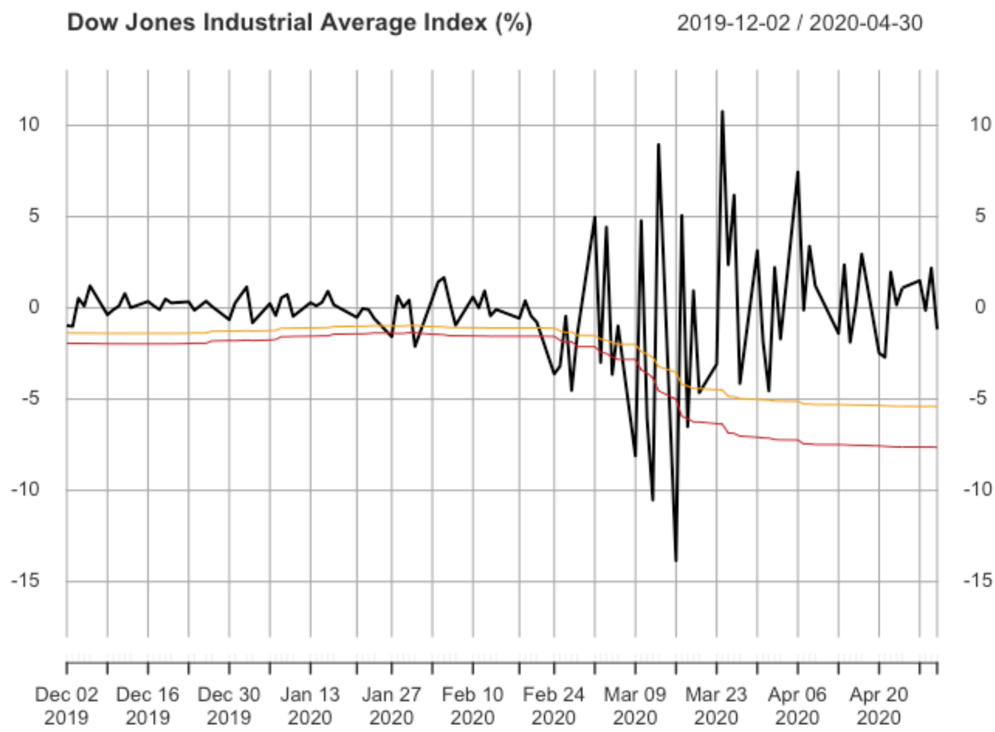
7.2. Study of VaR methodologies behaviour in the current high volatility period due to the uncertainty caused by the Covid-19 worldwide expansion. Which model has performed better in market risk quantification under such particular circumstances?

The Covid-19 pandemic is not only the most serious global health pandemic since the 1918 Spanish flu, but is set to become one of the most economically costly pandemics in recent history. In fact, the outbreak of the Coronavirus disease, which was originally detected on December 1st 2019 in the city of Wuhan (China), has had severe consequences for the worldwide economy. The current crisis is unprecedented since it combines a fall in global demand, tighter financial conditions and a major supply shock (World Bank, 2020). In fact, literature evidence revealed that the Covid-19 had a significant impact on the global financial markets. For instance, the Dow Jones and S&P indexes have experienced their biggest one-day drop since 1987. In that sense, according to the Financial Stability Board, “the global financial system faces the dual challenge to sustain the flow of credit amidst declining growth and to manage heightened risks”. This exogenous shock clearly placed the global financial system under strain.

Continued downward revisions of economic growth expectations and increased risk aversion, combined with high uncertainty about the future development of the pandemic, have led to a period of extremely high levels of volatility (FSB, 2020). In that respect, to conclude with the comprehensive review of the most used methodologies to compute value at risk, we will here after study the behaviour of each risk quantification approach of VaR under these exceptional conditions in order to examine which methodology performed better in this high volatility period and check if these results are in line with the previous ones or not.

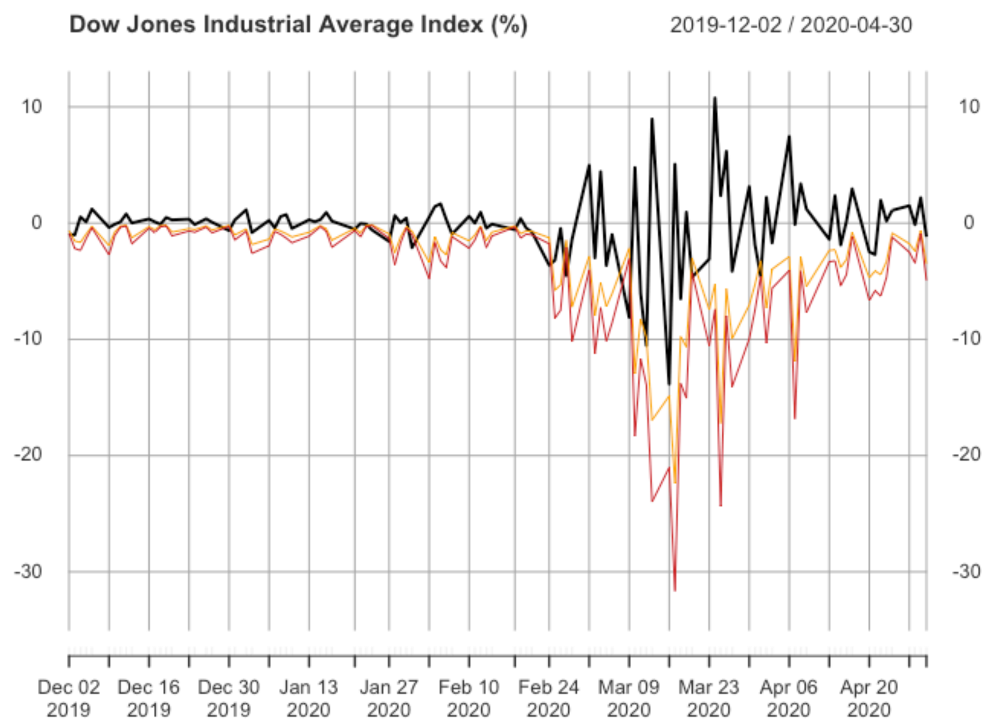
The following graphs show the obtained Dow Jones´s Index VaR results over this five-months period going from the 1st December 2019 to the 30th April 2020. The methodologies that have been considered are the same as before, i.e. Moving Average Volatility, Exponential Weighted Moving Average Volatility, GARCH (1,1) Model under Normal distribution assumption, GARCH (1,1) Model under Student-T distribution assumption, Historical Simulation, and Extreme Value Theory.

Graph 1: VaR results for the Moving Average Volatility Model



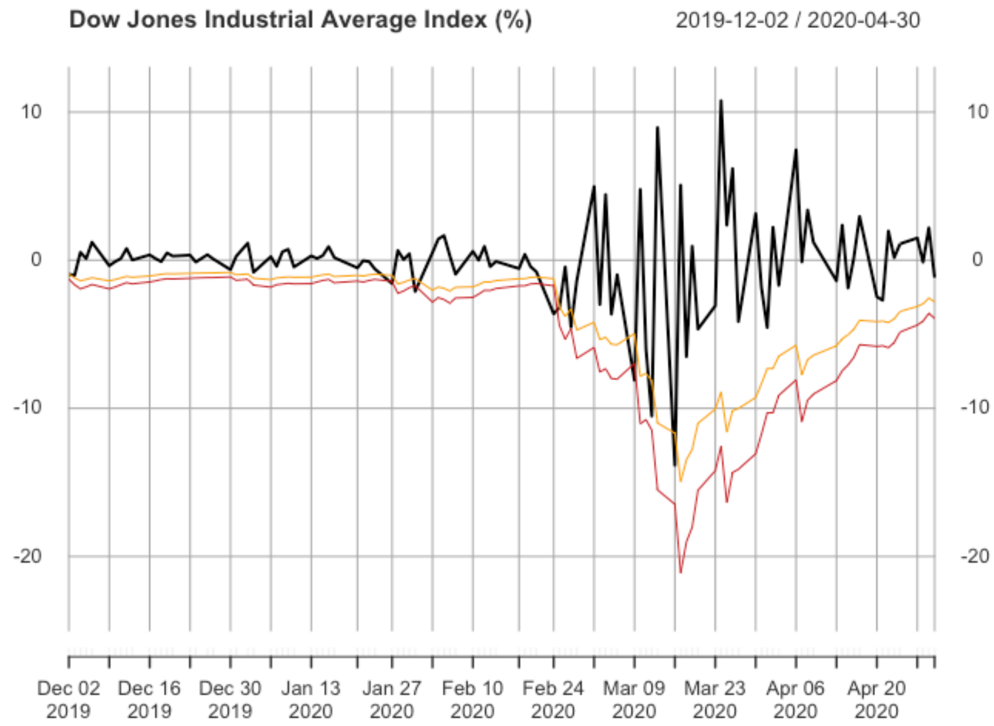
Daily returns appear in black, 95% confidence VaR in orange and 99% confidence VaR in red.

Graph 2: VaR results for the Exponential Weighted Moving Average Volatility Model



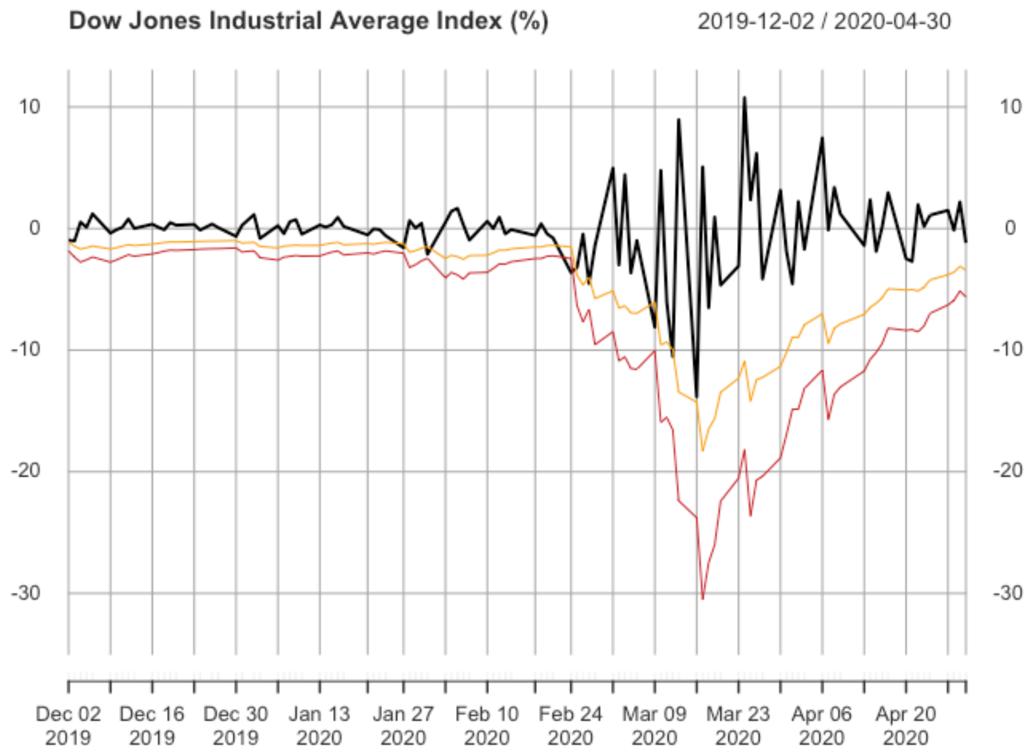
Daily returns appear in black, 95% confidence VaR in orange and 99% confidence VaR in red.

Graph 3: VaR results for the GARCH (1,1) Model under the Normal distribution assumption



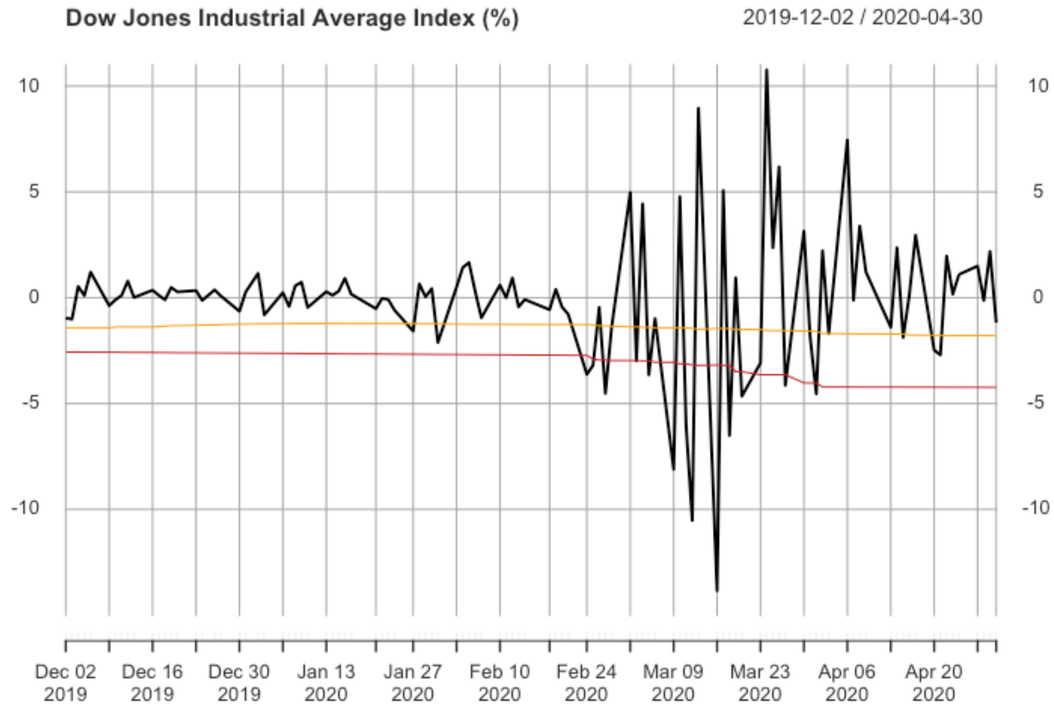
Daily returns appear in black, 95% confidence VaR in orange and 99% confidence VaR in red.

Graph 4: VaR results for the GARCH (1,1) Model under the Student-T distribution assumption



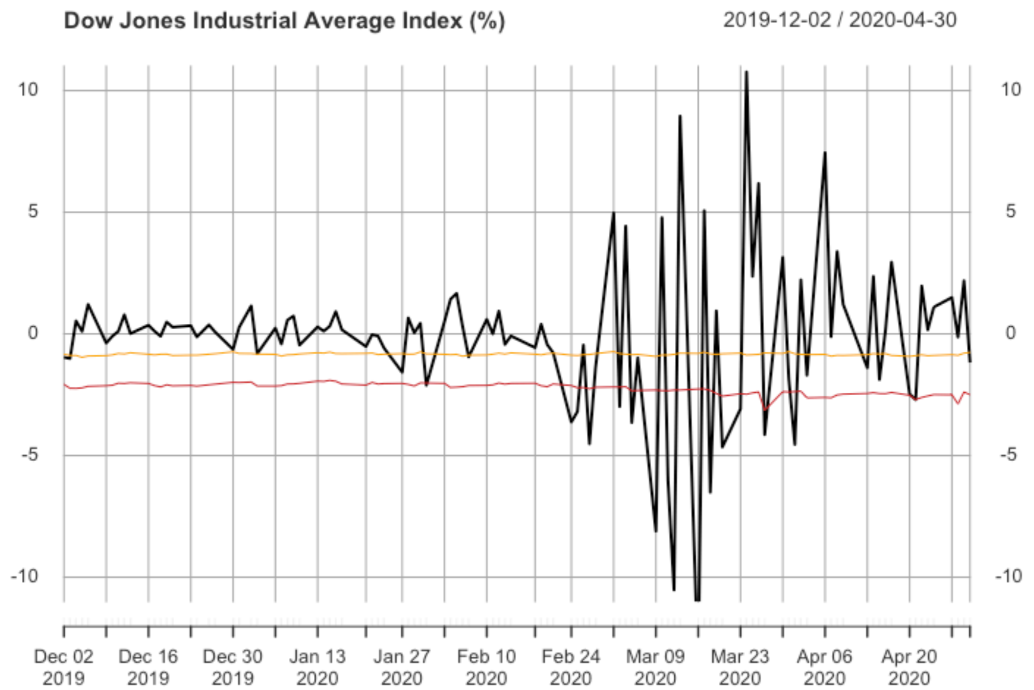
Daily returns appear in black, 95% confidence VaR in orange and 99% confidence VaR in red.

Graph 5: VaR results for the Historical Simulation Model



Daily returns appear in black, 95% confidence VaR in orange and 99% confidence VaR in red.

Graph 6: VaR results for the Extreme Value Theory Model



Daily returns appear in black, 95% confidence VaR in orange and 99% confidence VaR in red.

Summary of Results with the observed exceedances:

	Number of exceedances	
	95% Confidence Level	**99% Confidence Level**
Moving Average	13	12
EWMA or Riskmetrics	15	10
GARCH (1,1) Normal Distribution Assumption	9	4
GARCH (1,1) Student-T Distribution Assumption	6	1
Historical Simulation	22	13
Extreme Value Theory	27	16

Kupiec Test Results:

95% Confidence Level	
Expected Exceptions	5.2
Critical Value	3.8415

Methodology	Observed Exceedances	LR	Conclusion
Moving Average	13	8.8562	Reject H_0
EWMA	15	13.1876	Reject H_0
Garch (1,1) Normal	9	2.4223	Do not Reject H_0
Garch (1,1) Student-T	6	0.1237	Do not Reject H_0
Historical Simulation	22	32.8988	Reject H_0
Extreme Value Theory	27	50.5566	Reject H_0

99% Confidence Level	
Expected Exceptions	1.04
Critical Value	6.6349

Methodology	Observed Exceedances	LR	Conclusion
Moving Average	12	37.9869	Reject H_0
EWMA	10	28.1507	Reject H_0
Garch (1,1) Normal	4	4.9425	Do not Reject H_0
Garch (1,1) Student-T	1	0.0016	Do not Reject H_0
Historical Simulation	13	43.1954	Reject H_0
Extreme Value Theory	16	59.8351	Reject H_0

Analysing the results obtained we discern the great difficulty the models have had in general to adapt to this stress scenario in the market. In fact, the null hypothesis that the frequency of tail losses is equal to the significance level is rejected in four of the six studied models, for both 95% and 99% confidence levels.

- First, as we saw in the previous situation, which did not present this uncommon variability in the market, the Extreme Value Theory model has underestimated the risk in excess. It is the methodology that showed the worst results and we therefore can reject the null hypothesis with a higher level of certainty

- Second, contrary to the pre-pandemic situation, the results for the Historical Simulation and Moving Averages models here suggest that these methodologies do not grant a high degree of accuracy, especially in a context of 99% confidence level. In that sense, despite their acceptable results in a relatively stable period, these approaches have proved a very poor capacity to adapt to unstable scenarios in the market.

- Third, even if the Exponential Weighted Moving Average Volatility model still does not satisfactorily meet the needs of an acceptable model, in the face of greater uncertainty and instability its performance has not worsened.

Only the GARCH methodology with its two distributive variants seem to be properly fitted according to the Kupiec´s test. In fact, the null hypothesis cannot be rejected for either 95% or 99% confidence levels, suggesting that this approach efficiently captured the dramatic effects of the Covid-19 pandemic on global financial markets and so displayed a particularly interesting adaptive capacity to this new extreme situation. Singularly, the added value of the GARCH (1,1) under the assumption of a Student-T distribution needs to be highlighted. In fact, this model has been able to rigorously estimate the shape of the future returns and performed the best at both confidence levels, and the Kupiec test statistic value is way below the critic value.

8. CONCLUSION

Tragic events such as the 2008 global financial crisis precipitated, apart from the discernible economic consequences, a major breakdown in the global perception of risk and investors decision-making process. Accordingly, the so-called risk culture movement, which claimed for a stronger regulatory scheme of the financial system, began. In such circumstances, risk management field importance significantly increased in every world financial institution.

Among all the previous existing procedures to quantify the different types of risk in financial markets, the use of Value at Risk (VaR) became extremely popular since his introduction in the 1990s due to its simplicity in its appliance to estimate market risk. However, since the 2008 collapse, debates around its effectiveness to properly quantify risk in the more sophisticated financial markets were more and more frequent. This risk measurement technique could be defined as the maximum potential loss in value of a portfolio due to adverse market movements, for a given probability. Nevertheless, despite the fairly simple mathematical theory behind the VaR concept, there are nowadays several different approaches to compute this value, each of which include different assumptions about the financial series behaviour. The evaluation of some of these different methodologies has precisely been the core of this research.

In that respect, we can distinguish between parametric, non-parametric and semi-parametric VaR approaches. First, parametric methods rely on distributional assumptions of financial returns such as the Normal or the Student-T distributions. Within this category we have studied the Moving Average model, which shows progressive changes in variance; the Riskmetrics approach, which is based on an IGARCH (1,1) model that does take into account the clustering volatility phenomenon; and finally the more complex GARCH (1,1) model. Second, non-parametric methods do not depend on strong theoretical assumptions of the underlying properties of the data and so can accommodate wide tails, skewness or other any feature's in financial observations. Nonetheless, the non-parametric method that has been studied here, that is, the historical simulation model, depends on the strong assumption that the future will be quite similar to the recent past. In fact, this approach simulates different possible scenarios in search of an accurate risk estimation, but this simulation is based on historical data. Third, among the so-called semi-parametric VaR methodologies, which are considered to be a combination of the two previous types, we only focused on the fairly recent model that used the Extreme Value Theory to calculate value at risk. This branch of statistics applied to risk management focuses on the evaluation of the occurrence volatility of events or values more extreme than those observed previously in the given sample of a concrete random variable. And so, this approach tries to solve the issue of VaR, whose efficiency under extreme conditions has been clearly questioned.

In order to assess the overall performance of these five different methodologies we decided to compute the daily 95% and 99% confidence VaR of the well-known Dow Jones Index and carried out our research under two different scenarios: a more stable period going from November 2018 to November 2019, and a highly volatile period caused by the Covid-19 pandemic going from December 2019 to April 2020. A quantitative and qualitative analysis of results was done, for which we used the obtained graphs in R and the results of the Backtesting Kupiec Test.

The results of our study suggest that the GARCH (1,1) model, especially the one based in a Student-T distribution of financial returns, presents a more accurate estimation of market risk. In fact, this model clearly stands out in both low and high volatility situations for 95% and 99% VaR confidence levels. The better results of the GARCH (1,1) model under the Student-T assumption than under the Normal assumption can be explained by the fact that financial series distributions present heavy tails. Furthermore, despite there were significant differences among the remaining approaches, none of them did satisfactorily fulfilled the minimum requirements to be considered as a good methodology. In fact, even if historical simulation or moving averages estimations were acceptable under a relatively stable period, these two methodologies clearly showed a lack of adaptability to an extreme situation such as the Covid-19 crisis. This observation is not very surprising since we know that both models don´t take into account the clustering volatility phenomenon and thus don´t give higher weight to the most recent available data. Finally, both the Riskmetrics traditional approach and the Extreme Value Theory significantly underestimated risk in every period.

To conclude, although this study aims to highlight the GARCH (1,1) model as the most efficient approach to compute value at risk, an application of the several existing methods to calculate VaR is strongly recommended, since this could really give us a global idea of the ranges that VaR could handle. Undoubtedly, this value at risk measure is an extremely usable and useful way to estimate market risk, but there is much more room for improvement.

Finally, we would like to mention that this research has potential limitations. First of all, our results are only based on the studied Dow Jones Index which, even if being one of the most followed financial indexes, its behaviour does not necessarily correspond to the one of other indexes, individual assets or investment portfolios. Second, only the one-day time horizon VaR has been studied here. For further research, it could be interesting to expand our analysis to other time horizons such as weekly or monthly VaR and see whether the results vary or not. Third, a more in-depth analysis of the EVT approach would have been desirable, since other literature studies show significantly different results than the ones obtained here. A residual analysis of the model could have been done as a first step to try to find an explanation to this issue. Besides, we here only focused the Maximum Likelihood method to estimate the parameters of the limit distribution of financial returns, whereas there are many other ways to estimate them, such as the increasingly used Peaks Over Threshold method. Finally, we should keep in mind for eventual future work that there are other existing methodologies to compute value at risk that haven´t been mentioned nor studied here. These are for instance, the very used Montecarlo Simulation or the less frequent CaViaR method.

9. APPENDIX

This research relied to a great extent in the programming involved in both the collection and analysis of financial data. Every stage of VaR calculation, analysis and presentation of results was done within the R programming language environment. This appendix provides the basic codes needed for computing value at risk through all the studied approaches.

The initial step clearly involves the collection of the data, which is directly done using the *getSymbols* function of R *quantmod* package. This function is a wrapper to load data from various local or remote sources. In this case, since we wanted to download the Dow Jones Index returns time series data from an external source. In that sense, the argument *src* allowed us to set the source of the data: Yahoo finance.

```
# Importing data from Yahoo Finance:
DwJones <- getSymbols ("^DJI", from = "", to = “”, src ="yahoo", auto.assign = F) [,6]
```

Once the 3-years historic data was recorded in our R environment, every subsequent stage of the coding needed for our research was done using a similar methodology. First, recall that the main goal here was to compute the 1-day time horizon value at risk over a one financial year period for a specific level of confidence. Therefore, the key idea while creating the several codes in R was to generate one single VaR function for each methodology that will directly compute the 1-day value at risk given a certain number of parameters. Then, once this function was generated and verified, we simply had to create a loop using the *for* function so that VaR will be computed for the desired number of days. Here below is presented the loop needed for every different VaR calculation. Also, note that two matrixes were created in order to store the consecutive VaR results, one for each respective confidence level. For each matrix, the number of columns and rows needs to be specified using the *nrow* and *ncol* arguments.

```
# Creating two matrixes where we will put the VaR 95% and 99% confidence results:
Backtesting.VaR.95 <- matrix (nrow = 250, ncol = 1)
Backtesting.VaR.99 <- matrix (nrow = 250, ncol = 1)

# Loop for VaR calculation the next 250 days:
for (i in 1:250) {
Backtesting.VaR.95 [i] <- VaR.function (DwJones[i:757+i], p, n)


for (i in 1:250) {
Backtesting.VaR.99 [i] <- VaR.function (DwJones[i:757+i], p, n)
}
```

As a matter of fact, the difference in each particular code for the several VaR calculation methodologies resides in the specific VaR function used. In each case, the diverse R tools and libraries needed are presented and explained hereunder.

1. Moving Average Volatility Model

Through the moving average volatility model, the variance is simply calculated as a moving average of the squared returns of the financial time series of n observations. As a vital step, we must first perform the returns calculation. In order to do so, we could simply take the ratio between the current price versus the last price and calculate its napierian logarithm, or rather calculate the log differences, which is more straightforward according to the log properties.

In addition, for future steps, it is important to clean the data by removing the non-available results that appear in the created vector, as it's the case with the first element. Two possibilities may exist here. First, since in this particular case we know that the only missing value is the first one, the simplest solution would be to simply remove the first element by using R basic programming tools, that is, inserting a [-1] after the returns calculations. As an alternative, a more general solution could be to apply the *na.omit* command from *timeSeries,* which allows us to remove all NA results from the created vector, just in case there were other unnoticed missing values among the initial data frame. Second, after deriving the squared returns and cleaning the vector, the available *SMA* function from the *fTrading* package computes the desired variance. This function calculates the arithmetic mean of the series over the past n observations. That is, the way it is set, it actually computes the arithmetic mean of the volatility of returns, which is precisely what we want. And so, since we only need the last result of the volatility of returns, we use the function *length* to select it.

Lastly, the normal distribution z-score is calculated through the *qnorm* function of the *stats* library. Note that this *qnorm* function that gives the 95% and 99% confidence z-scores will be also used in the other studied parametric methods, which also rely on the normality assumption of returns. As a result, this VaR calculation model can be easily programmed in R as follows:

```
# MA VaR function:

VaR.MA <- function (data, p, n) {
rend <- diff (log(data))
rend.def <- na.omit (rend)
var.MA <- SMA (rend.def^2, n = n)
se.MA <- (var.MA[length(var.MA)])^(1/2)
VaRMA <- -qnorm (p)*se.MA
return (VaRMA=VaRMA)
}
```

2. Exponential Weighted Moving Average Volatility or Riskmetrics Model

In contrast with the previous moving average volatility model, Riskmetrics approach gives higher weights to recent observations than past observations so that the volatility clustering phenomenon observed in financial time series is taken into account. In that sense, volatility forecasts are estimated through exponentially weighted moving averages, considering a 0.94 corrective weighting factor as recommended by Riskmetrics. The *emaTA* function from the *fTrading* library precisely does that work for us. In fact, this *emaTA* function is very similar to the previous *SMA* except that, in this case, it calculates an exponentially-weighed mean, giving more weight to recent observations. However, the logic of the procedure very similar to the one presented below, the only difference being the function used to compute volatility. The VaR function code is explicitly presented here below:

```
# EWMA VaR function:

VaR.EWMA <- function (data, p, lambda) {
  rend2. <- diff (log(data))
  rend.def2 <- na.omit (rend2.)
  var.EWMA <- emaTA (rend.def2^2, lambda = lambda)
  se.EWMA <- (var.EWMA[length(var.EWMA)])^(1/2)
  VaREWMA <- -qnorm (p)*se.EWMA
  return (VaREWMA=VaREWMA)
}
```

3. GARCH (1,1) Model of volatility estimation

The more sophisticated econometric approach using generalized autoregressive conditional heteroskedastic processes to model volatility is performed in R using the *garchFit* function contained in the *fGarch* package, which estimates the conditional mean and conditional variance of the time series returns r_t for a GARCH (1,1) model. Once these estimators are obtained, the following step is to derive the 1 step-ahead forecasts for both, the conditional mean and the conditional variance. This can be effected by using the *predict* function, which allows us to predict the n step-ahead estimations for any *fGarch* object. Then, VaR derivation is automatic:

```
# GARCH VaR function:

VaR.GARCH <- function (data, p) {
  rend3. <- diff (log(data))
  rend.def3 <- na.omit (rend3.)
  res <- garchFit (formula = ~ garch (1, 1), data = rend.def3,
cond.dist = "norm")
  cond.var.Garch  <- predict (res, n.ahead = 1)[1,3]
  cond.mean.Garch  <- predict (res, n.ahead = 1)[1,1]
  VaR <- cond.mean.Garch  - qnorm (p)*cond.var.Garch
  return (VaR=VaR)
}
```

Note that the previous code corresponds to VaR calculation through a GARCH (1,1) model estimation of volatility under the Normal returns distribution assumption. Alternatively, if we wanted to calculate VaR under a Student-T returns distribution we should substitute the Normal distribution z-scores computed with the *qnorm* function by the Student-T z-scores that could be easily derived using the *qt* command for a specified degrees of freedom level.

4. Historical Simulation VaR

The VaR calculation process though the historical simulation approach is fairly simple. All we need is to create a function which computes the daily price returns. In the generated vector, the first value will be logically missing, and so It is important to remove it for cleaning purposes. Finally, we must take the corresponding percentile depending on the desired level of significance. This work is done by the *quantile* function of the *stats* library.

```
# HS VaR Function:

VaR.historic <- function (price, p) {
  returns <- diff(log(price)) [-1]
  HistoricSimulationVaR <- quantile (returns*100, c(p))
  return (HistoricSimulationVaR=HistoricSimulationVaR)
}
```

5. Extreme Value Theory Model

Lastly, we compute value at risk through the Extreme Value approach. After calculating the returns and removing the first element as in the previous methods, the limit distribution parameters are estimated using the Maximum Likelihood Method. The *gev* function obtained with the *evir* package estimates these shape, scale and location parameters of the tails distribution function for a given number of subsamples n that must be specified. Note that we had to use the *as.numeric* command before being able to estimate these parameters as, for some reason, the *gev* function had problems while interpreting the previously generated returns vector as numbers. Once this problem was solved, we could finally estimate the parameters. In a final step, we selected them by using h$pars.ests [1], h$pars.ests [2], and h$pars.ests [3] respectively, and applied the estimations to the VaR formula under EVT.

```
# EVT VaR function:

EVTVaR <- function (price, n, p) {
returns <- -diff(log(price))*100
l <- as.numeric (returns[-1])
h <- gev (l, block=n)
xi <- h$par.ests [1]
sigma <- h$par.ests [2]
mu <- h$par.ests [3]
v <- 1- (-n*log(1-p))^(-xi)
EVTVaR <- mu - (sigma/xi) * v
return(EVTVaR=EVTVaR)
}
```

10. REFERENCES

- Abad, P., Benito, S., & López, C. (2013). A comprehensive review of Value at Risk methodologies. *The Spanish Review of Financial Economics*, (Vol. 15. Issue 2.), 41-90.
- Alonso, J. & Semaán, P. (2010). Cálculo del VaR con volatilidad no constante en R. *Apuntes de Economía No. 22.* Universidad Icesi, Colombia.
- Alonso, J. & Semaán, P. (2009). Cálculo del Valor en Riesgo y Pérdida Esperada mediante R: Empleando modelos con volatilidad constante. *Apuntes de Economía No.21.* Universidad Icesi, Colombia.
- Artzner, P., Delbaen, F., Eber, J., & Heath, D. (1997). Coherent Measures of Risk. *Mathematical Finance*, (Volume 9, Issue 3), 203-228.
- Claro, F., Contador, S., & Quiroga, C. (2006). *Teoría del Valor Extremo: Aplicación de la teoría al Índice NASDAQ Periodos: 28/Octubre/1996 a 26/Octubre/2006* (Pregrado). Universidad de Chile.
- Delfiner, M., Balzzarotti, V., & Del Canto, Á. (2001). *Backtesting: Performance of capital requirements for market risk in the BCRA*. University Library of Munich, Germany.
- Dowd, K. (2002). *Measuring Market Risk* (2nd ed.). John Wiley & Sons.
- Ernst & Young. (2013). *Remaking financial services, five years after the financial crisis*. Presentation.
- Franco Arbeláez, Luis Ceferino, & Franco Ceballos, Luis Eduardo (2005). El valor en riesgo condicional CVaR como medida coherente de riesgo. Revista Ingenierías Universidad de Medellín, 4(6),43-54.
- García Domínguez, C., Meza González, A., & Ventura García, B. (2017). Methods Of Calculation Value At Risk (Undergraduate). Universidad de La Laguna.
- Gnedenko, B. (1943). Sur la distribution limite du terme maximum d'une série aléatorie. Annals of Mathematics, (44), 423-453.
- Gustafsson, M. & Lundberg, C. (2009). An empirical evaluation of Value at Risk. University of Gothenburg, Sweden.
- Hill, B. (1975). A Simple General Approach to Inference About the Tail of a Distribution. *Annals Of Statistics*, (Volume 3, Number 5), 1163-1174.
- Jenkinson AF. The frequency distribution of the annual maximum (or minimum) of meteorological elements. Q J R Meteorol Soc 1955; 81: 158–171.

- Jorion, P. (2001). *Value at Risk: The New Benchmark for Managing Financial Risk.* (2nd ed.). [New York]: McGraw-Hill.
- J.P. Morgan/Reuters (1996). Riskmetrics, Technical Document. New York, Fourth Edition.
- Kirman, A. & Teyssiere, G. (2005). Volatility Clustering in Financial Markets: Empirical Facts and Agent-Based Models. *Long memory in economics.*
- Kupiec, P. (1995). Techniques for Verifying the Accuracy of Risk Measurement Models. *The Journal Of Derivatives*, (3), 73-84.
- Li, W. (2016). Value at Risk and its calculations: an overview. Retrieved from http://web.mst.edu/~huwen/teaching_VaR_Weiqian_Li.pdf
- Manganelli, S. & F. Engle, R. (2001). Value at Risk Models in Finance. Working Paper No.75. European Central Bank.
- McNeil, A. (1998). *Calculating Quantile Risk Measures for Financial Return Series using Extreme Value Theory*. ETH Zentrum.
- McNeil, A., & Frey, R. (2000). Estimation of tail-related risk measures for heteroscedastic financial time series: an extreme value approach. *Journal Of Empirical Finance*, (Volume 7, issues 3-4), 271-300.
- Nieppola, O. (2009). Backtesting Value at Risk Models. Helsinki School of Economics.
- NYU Stern School of Business. (2014). Value at Risk (VaR). New York. Retrieved from http://people.stern.nyu.edu/adamodar/pdfiles/papers/VAR.pdf
- Pickands, J. (1975). Statistical Inference Using Extreme Order Statistics. Annals of Statistics, (Volume 3, Number 1), 119-131.
- Roszkowski, M. (2010). Risk Perception and Risk Tolerance Changes Attributable to the 2008 Economic Crisis: A Subtle but Critical Difference. *Journal Of Financial Service Professionals*.
- Tsay, R. (2013). An introduction to Analysis of Financial Data with R (pp. 327-287). New Jersey: John Wiley & Sons.
- Wiener, Z. (1997). Introduction to VaR. *Risk management and Regulation in Banking.* The Hebrew University of Jerusalem, Jerusalem.